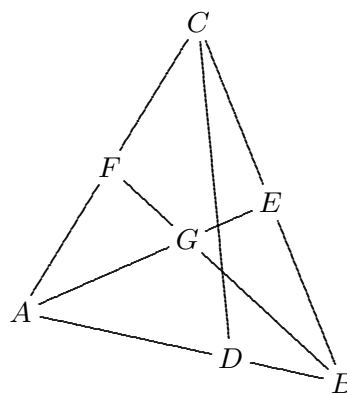


Winggeom 2D Introductory Activities

0. Start *Winggeom* by double-clicking its icon. Click the menu item **Window|2-dim**. This will create a small drawing window. From now on, you will be using its menu bar. Items in boldface are menu items or dialog buttons that are meant to be clicked with the mouse. Keyboard input, such as *Enter*, *Escape*, and *Ctrl+W*, is written in italics.

1. The left and right mouse buttons perform different functions, depending on the item that is checked on the **Btns** menu. The primary drawing mode is **Btns|Segments**. Check this item by clicking it, then point at three random places in the drawing window and click the *right* button. This labels three points A , B , and C . Point at A , hold down the *left* button, slide the pointer to B , and release the button. Segment AB now appears on the screen (unless the button release was not close enough to B). Repeat the process to draw segments BC and CA .

2. Right-click a point onto segment AB . Its assigned label will be D , because the program always selects the first available label for a new point. Right-clicking is one way of marking a point on a segment. Here is another: Click **Point|on Segment**, type the list BC, CA into the “relative to segment” box, notice that the “coordinate” box shows the value 0.5, and click **mark**. Labels E and F appear at the midpoints of segment BC and CA , respectively. Close this dialog box by pressing *Escape* or by clicking **close**. Use the left button to draw segments CD , AE , and BF , as in step 1. Point at the intersection of AE and BF and click the right button. The intersection point should now be labeled G , as shown.



3. Click **Btns|Drag vertices** to put the mouse into a different mode. Point at B , hold down the left button (notice that the label color changes), and slide the mouse. Point B moves, while everything else in the figure adjusts its position accordingly. In particular, E maintains its status as the midpoint of segment BC , and D holds its relative position on segment AB . These adjustments continue until you release the button. This is called *dragging* point B . Points A and C can also be dragged. If you try to drag either E or F , however, the whole triangle moves rigidly. Think about why this happens. If you try to drag D , you will find that it does move — but only along the segment it was placed on. Notice that the intersection G of the medians AE and BF is not usually on segment CD .

4. Click **Meas** to open a new dialog box. The cursor is blinking in an edit box. Type the ratio AD/AB into the box (the program does not distinguish between upper and lower case letters, by the way) and press *Enter*. The current value of this ratio is shown in both the dialog box and the drawing window. This is because the measurement dialog must be closed (just press *Escape*) before drawing operations can resume. Return to dragging point D along AB . The displayed value of AD/AB tells you the exact position of D on the segment. Notice the value of this ratio when G seems to be on segment CD . Is it what you would expect?

Winggeom 2D Introductory Activities

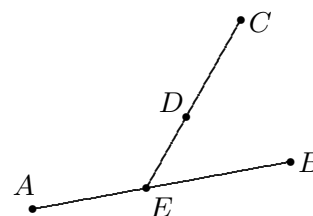
5. Not only does the program assign labels to new points, it also centers the labels directly over the points, and it puts measurements in the upper left corner of the drawing window. You can override these choices. First click **Btns|Text** to put the mouse into a new mode. Point at label A and press the right button. A new dialog box allows to change the label. Type P into the box and press *Enter*. You will see label A change to P . In the same way, change label B to Q . Notice that the displayed ratio AD/AB is now “undefined”, so click **Meas**, select AD/AB in the list box, and click **delete**. Type PD/PQ , press *Enter*, and press *Escape*. Next point at any label in the figure, hold down the left button, slide the mouse a short distance, and release the button. The point does not move but the label does. Notice that the point is marked by a small circle, usually hidden by the label. Click **Edit|Labels|Offset** to move all the labels (this seldom places every label satisfactorily, however). Click **Edit|Labels|Home** to recenter all the labels. To choose a different font for the labels, click **Edit|Labels|Font**.

6. When the mouse is in **Btns|Text** mode, the right button is used to insert new text or edit old text anywhere on the screen, and the left button is used to drag (reposition) text. For example, drag the measurement to a spot that is closer to segment PQ . To send all calculations back to their default positions, click **Other|Measurements|Home**.

7. Click **File|New** to start anew. When asked to save your work, respond **no**.

8. Put the mouse into **Btns|Segments** mode, right-click four new points, then use the left button to create segments AB and CD . Put the mouse into **Btns|Drag vertices** mode, and drag some of the points so that the segments intersect. Here is a new way of labeling the intersection that is occasionally useful: Click **Point|Intersection|Line-line**, type AB into one edit box, CD into the other, and press *Enter*. Label E should appear. Press *Escape* to close the box.

9. Make sure that **Other|Autoextend** does not have a check mark. (If necessary, click it to remove the check.) Drag B somewhere so that segments AB and CD no longer intersect, and watch E disappear. Now activate the **Other|Autoextend** feature by clicking it. Notice that the segments are now extended as much as necessary to show their intersection, regardless of where the points are dragged.



10. Click **Edit|Undo** (or press *Ctrl+Z*). This undoes the most recent construction step, so intersection E is no longer labeled. Click **Edit|Redo** (or press *Ctrl+Y*) to redo the last step. You can always click **Other|Lists|History** to see a step-by-step description of the current figure. (The text display does not change if you do anything to the drawing, however. To update the contents, you must click **close** and re-open the window.)

11. Make a fresh start by clicking **File|New**, then click **Point|Coordinates** to open a coordinate-entry dialog box. Type 3 into the x box, 1 into the y box, and press *Enter* (or click **mark**). Point $A = (3, 1)$ should appear. In the same way, label the points $B = (5, 2)$ and $C = (6, -5)$. Close this dialog box. Click **View|Axes|Axes** (or press *Ctrl+A*) to make the coordinate axes disappear. Make them reappear in the same way.

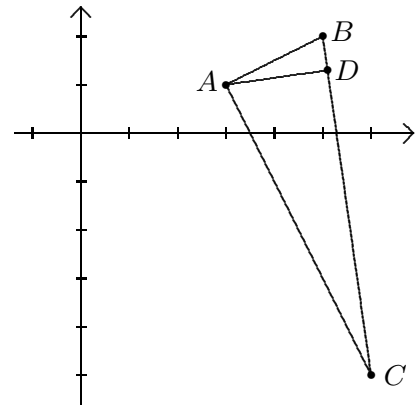
Winggeom 2D Introductory Activities

12. The mouse is probably still in **Btns|Drag vertices** mode. Notice that you cannot drag any of the points without moving the whole sheet of graph paper. This is because the points were defined to keep them in one place. Now put the mouse into **Btns|Segments** mode and finish drawing triangle ABC .

13. Click open the **Meas** dialog box. Type $\angle ABC$ into the edit box and press *Enter*. The size of angle ABC is displayed. The inequality symbol tells the program that an angle size is requested. If you try to describe the same angle as just $\angle B$, the program will not understand, for it wants a three-label description. Type ABC into the edit box and press *Enter* to see the area of triangle ABC . Type $AB + BC + CA$ and press *Enter* to see the perimeter of triangle ABC . To deselect the highlighted perimeter in the dialog's list box, click it. Then click the angle measurement to select it. Clicking the **hide** button makes the measurement disappear from the screen, and clicking **show** makes it reappear. Press *Escape* to close the dialog box.

14. If you want dimensional information appended to displayed measurements (to designate length, area, and angles), the menu item **Other|Measurements|Units** must be checked. Until the screen is forced to refresh itself, nothing happens after you change the status of this item.

15. Click **Line|Perpendiculars|Altitude**, type BC into the “perp to line” box, A into the “from point” box, and click **draw**. Segment AD appears, constructed so that D is on segment BC and angle ADB is a right angle. Press *Escape* to close the dialog box.



16. Open the **Meas** dialog box again. Type D and press *Enter*. You should see that the coordinates of D are $(5.1, 1.3)$. To confirm that AD does intersect BC perpendicularly, type $\angle ADB$ and press *Enter*. Press *Escape* to close the dialog. Click **Other|Lists|Lines** to open a text window that enumerates all combinations of collinear points in the figure. If the coordinate axes are showing, a Cartesian equation is displayed for each line, as well as its slope. Click **close** to hide this text window.

17. If you would prefer to have the mouse functions more visible (instead of hidden in a menu), click **Btns|Toolbar**. This small, movable dialog box displays the current mouse function, and it gives you another way to alter it. Put the mouse into **Drag vertices mode**. As you noticed earlier, dragging any of the labeled points simply slides the sheet of graph paper across the screen. Do so now, and try to make some of its vertices disappear from view (if your drawing window does not fill the screen, you can make the entire triangle disappear). To quickly restore any figure to the center of the screen, so that all of its parts are visible, click **View|Window** (or press *Ctrl+W*). This also repositions any measurements.

Winggeom 2D Introductory Activities

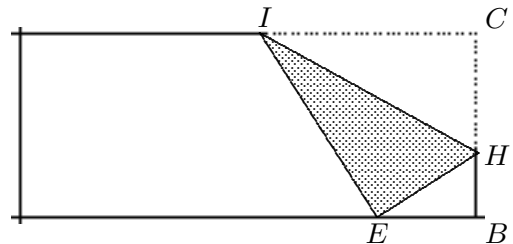
18. Click **File|New** to begin a new figure. If the coordinate axes are still showing, press $Ctrl+A$ to turn them off. Click **Units|Polygon|Parallelogram**, type 12 into the first “side” box, change the “angle” to 90, and change the second “side” to 4. When you press *Enter* (or click **ok**), a rectangle $ABCD$ should appear. Because it is three times as wide as it is tall, it might not fit well in the current window, but you can change the size and shape of a window by dragging its border. Put the mouse into **Btns|Segments** mode and right-click a point E onto the segment AB , closer to B than C is. Then use the left button to draw the segment CE .

19. Click **Line|Perpendiculars|Bisector**, type CE into the edit box, and press *Enter*. The bisector FG goes through the midpoint F of segment CE , and it should also intersect sides BC and CD . Use the right button to label these intersections, with H on BC and I on CD . Then use the left button to draw segments EH and EI .

20. Time to clean up. Points F and G are no longer needed, so click **Edit|Delete|Point**, type FG into the box, and press *Enter*. Click **Edit|Delete|Line**, type the list CE, CH, CI into the box, and press *Enter*. To turn the bisector HI into a segment, open the dialog box **Line|Extensions**, type the list HI, IH , and press *Enter*. (The rays HI and IH are thereby turned off. Rays can be turned on in the same fashion.)

21. Use the left button to reconnect segments DI, IC, CH , and HB . The reason for this strange move will be explained next. Click **Edit|Highlights|Line attributes**, type the list CH, CI into the edit box, click **style** until “dotted” appears, and click **apply**. Because the program was told to forget that C, I , and D are collinear (this was the reason for erasing segment CD and then redrawing it in two pieces), the dotted style does not apply to DI . Close the dialog box. Click **Edit|Highlights|Fill**, open the **color** window, click the red cell, click **polygon**, type EHI into the adjacent box, and click **fill**. Triangle EHI should now be red. Close this dialog box, and press $Ctrl+W$ to center the drawing, which should now look like the illustration below.

22. This construction is meant to simulate the folding of a rectangular sheet of paper, so that one corner (C) is matched with a point (E) on another edge. The dotted segments mark where the paper used to be, before it was folded over, and the red color is found on the underside of the sheet.



23. Put the mouse into **Btns|Drag vertices** mode and make sure that the menu item **Other|Autoextend** is checked. Use the left button to slide E along side AB . Move the mouse slowly! If you slide E too close to B or too far from B , the construction will collapse, because H has to be on side BC , and I has to be on side CD .

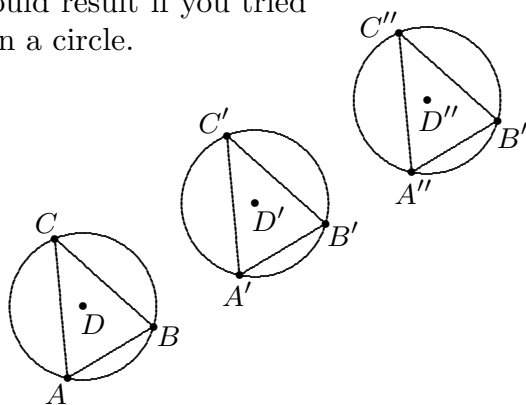
24. Open the **Meas** dialog and ask for $\angle BHE$ and $2 < HIC$ (one at a time, pressing *Enter* after each). *Escape* from the dialog box. Notice that the equality of these two measurements is unaffected by moving E along AB . Explain why this should be expected.

Winggeom 2D Introductory Activities

25. Click **File|New** to begin a new figure. (It is possible to have several drawings open at the same time — just click **2-dim** again on the main menu bar — but this is not necessary for this tour.) Click **Units|Random|Triangle**. This is a one-click method of putting three random points on the screen and connecting them with segments.

26. Click **Circle|Circumcircle**. The resulting dialog box probably already has ABC highlighted in the edit box, so just press *Enter*. The program constructs the circle that goes through all three vertices of the triangle. The center of the circle is labeled D . The program keeps track of each circle by recording its center and at least one point on the circle. Deleting the center of a circle would therefore require that the circle itself be deleted. To safeguard against this happening inadvertently, the program will not let you do it: Click **Edit|Delete|Point**, type D into the edit box, and press *Enter*. Click **ok** to close the error-message box. The same message would result if you tried to delete a point that was the only point marked on a circle.

27. Click **Transf|Translate**, type 2 into the “by the multiple” box, and AB into the “of vector” box (if it is not already there). Leave the “vertices” box alone, for it lists all the vertices currently in the figure, and that is the plan. To see the result of sliding the whole figure twice the length of vector AB , press *Enter*. The new figure consists of two triangles, two circles, and eight vertices. The four new vertices have been labeled by attaching primes to the original labels. Click **Transf|Last repeat** (or press $F7$). This applies the current transformation to its most recent images. The figure should now look like the illustration.



28. With the mouse in **Btms|Drag vertices** mode, drag vertex C around the screen, and notice what happens to all the images. Because the translation is defined in terms of segment AB , the effect is quite different if you drag B around the screen instead. Try it. Also notice that you can not drag points other than A , B , or C without dragging the entire figure rigidly. Why is this to be expected?

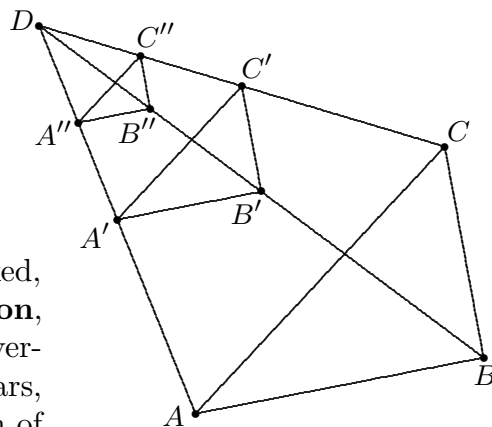
29. Click **Transf|Translation** again, type 2 into the “by the multiple” box, type AC into the “of vector” box, and leave the “vertices” box filled with the default list of all vertices. Press *Enter*, then press $F7$. You should now see nine triangles, nine circles and their centers, and thirty-six labels. The program labels transformed vertices by using subscripts and primes. Having this many labels on the screen at the same time can be confusing, so press $Ctrl+L$ (the shortcut for **Edit|Labels|Hide**) to turn all the labels off. (They can be turned on in the same way.) With the labels off, press $Ctrl+D$ (the shortcut for **Edit|Labels|Dot mode**) several times. Notice that points are marked by open circles, closed circles, diagonal crosses, horizontal crosses, or nothing at all. With the open circles on the screen, click **Edit|Labels|Bullet size**, type a one-digit number into the box, and press *Enter*. The default size for the circles is 2, but you may prefer a different size (and the program will remember).

Winggeom 2D Introductory Activities

30. Click **File|New** to begin a new figure. Click **Units|Random|Triangle** again. With the mouse in **Btms|Segments** mode, right-click a random point D onto the screen, well away from triangle ABC . Open the **Transf|Dilate** dialog box, which is used to define both rotations and dilations (or combinations of the two). It is now configured for a simple dilation, which means that 0.0 shows in the “angle” box. Type D into the “center” box, ABC into the “vertices” box, and press *Enter*. Press *F7* once to apply the dilation again.

31. Use the left button to draw segments DA , DB , and DC . Points A' and A'' will look like they also lie on segment DA . Because this property is indeed built into the definition of a dilation, you know that these points are collinear. Nevertheless, the points were not marked on the segment (which appeared later in the construction, anyway), and the program does not know any theorems of geometry, so it does not recognize that A' and A'' are in fact on DA . To see evidence of this, click **Edit|Delete|Line**, type AA' into the box, and press *Enter*. Notice that nothing disappears from the figure.

32. The figure should resemble the illustration. Put the mouse into **Btms|Drag vertices** mode, and drag the primary vertices (A , B , C , and D) around the screen. Notice that the response of the figure when D is moved is quite different from the response of the figure when A is moved.



33. If the menu item **Transf|Save labels** is checked, click it to remove the check. Click **Transf|Translation**, type AB into the “of vector” box, ABC into the “vertices” box, and press *Enter*. A new triangle appears, probably named $A_0B_0C_0$. According to the definition of vector translation, points A_0 and B have to coincide. Nevertheless, the program has assigned a new label to what is really the same point. The program thinks that the points are different — indeed, because A_0 had to be calculated, it might actually differ from B in the twentieth decimal place! This is a situation when you might feel obliged to override the program’s lack of understanding, and you can: Click **Edit|Undo** (or press *Ctrl+Z*) to undo the translation, click **Transf|Save labels** to turn this feature back on, then click **Edit|Undo undo** (or press *Ctrl+Y* to redo the translation. Notice that label A_0 does not appear this time. The program has been given permission to regard the nearness of A_0 and B as sufficient reason to identify these points in its records.

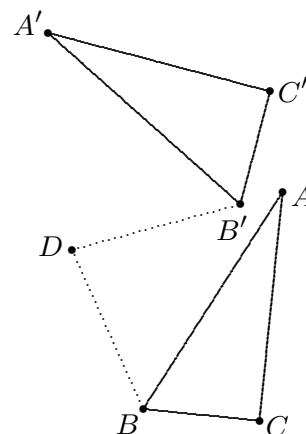
34. Click **File|New** to begin a new figure. Click **Units|Random|Right triangle** (notice the word *right*). Even though the vertices of this figure are randomly generated, they can not be dragged independently of each other. Try it — the triangle moves rigidly as a unit. This is because the program has been taught to respect the right-angle definition.

Winggeom 2D Introductory Activities

35. Put the mouse into **Btns|Segments** mode and right-click a random point D onto the screen, somewhere outside triangle ABC . Click **Transf|Rotation**, type D into the “using center” box, notice that the “dilation factor” is 1.0 (meaning no dilation), type ABC into the “vertices” box, and type $90\#$ into the “angle” box (you must include the symbol $\#$). Press *Enter* to see the result of rotating triangle ABC around the pivot point D . The size of the rotation angle depends on the value of the number $\#$. Before discovering its value in the next paragraph, estimate it by examining your figure.

36. Open the **Animate|# slider** dialog box, which displays the current value of $\#$. One way to change that value is to type a new value for the highlighted text, so type 0.5 into the edit box and press *Enter*. Notice that the figure changes as well. You now see the result of applying a 45-degree rotation, centered at D , to triangle ABC . Point at the arrow at one end of the scroll bar, and hold down the left button. The control will slide slowly, changing the value of $\#$, as well as the size $90\#$ of the rotation angle. You can also slide the control by dragging it. If you click **autoreverse**, the dialog box disappears but the bar moves back and forth invisibly. Notice the caption on the drawing window, which tells you to press the Q key to stop this animation. Watch for a while, then press Q .

37. Click **Line|Segments**, type BDB' into the edit box, and press *Enter*. The segments BD and DB' appear. Click **Edit|Highlights|Line attributes**, type BDB' into the edit box, click **style** until “dotted” appears, and press *Enter*. Press *Escape* to close this dialog box. Your figure should now resemble the illustration.



38. Open the **Meas** dialog box. Type $\#$ and press *Enter*. Type $90\#$ and press *Enter*. Type $\angle BDB'$ and press *Enter*. Close the dialog. Explain the coincidence of displayed values. Use the $\#$ -slider to vary the value of $\#$, and notice the changing screen display.

39. Click the **Transf|Save labels** item to enable it. Click **Transf|Rotation**, type D into the “using center” box, $A'B'C'$ into the “vertices” box, $90\#$ into the “angle size” box, then press *Enter*. Triangle $A''B''C''$ should appear. Press $F7$ four times. There should now be seven triangles on the screen, the last one being $A_3''B_3''C_3''$.

40. Type $2/3$ into the edit box that displays the current value of $\#$, and press *Enter*. Explain why there are now only six triangles on the screen. Where did $A_3''B_3''C_3''$ go? If you need a hint, type 0.6 into the edit box and press *Enter*.

41. Click **File|New** to begin a new figure. Click **Units|Segment**, and press *Enter* twice. A horizontal segment whose length is 1 (on the underlying graph paper) appears. Click **Line|Angles|New**, type AB into the “initial ray” box, 75 into the “angle size” box, and click **draw**. Ray AC appears. Now type BA into the “initial ray” box and -60 into the “angle size” box, and click **draw** again. Ray BD appears. Notice the use of a negative sign to describe the second angle. What positive angle would also have produced ray BD ?

Winggeom 2D Introductory Activities

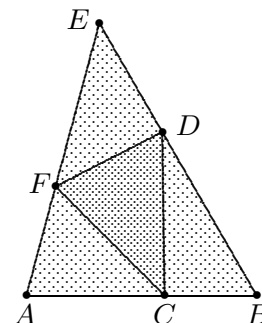
42. The intersection of these rays may well be outside the drawing window. One way to label this (perhaps invisible) point is to click **Point|Intersection|Line-line**, type AC and BD into the boxes, and click **mark**. The intersection E should now be visible. Press *Escape* to close the dialog box.

43. Triangle ABE has been constructed so that its angles are 75, 60, and 45 degrees. To finish the job, click **Edit|Delete|Point**, type CD into the box, and press *Enter*. Click **Line|Extensions**, type the list AE, BE into the box, and press *Enter*.

44. Type 0.6 into the edit box that displays the current value of $\#$, and press *Enter*. Then click the menu item **Point|on Segment**. This way of marking points on segments offers unlimited possibilities for animation and control. For example, type the list AB, BE, EA into the “relative to segment” box. If you were to click **mark** now, with 0.5 in the “coordinate” box, you would see all three midpoints of the sides of triangle ABE appear. Instead, replace the 0.5 by the symbol $\#$, which you know stands for the value 0.6, and press *Enter*. Point C appears on segment AB , 60% of the way from A to B , point D appears on segment BE , 60% of the way from B to E , and point F appears on segment EA , 60% of the way from E to A .

45. Put the mouse into **Btms|Segments** mode and use the left button to connect the vertices of triangle CDF . Notice that the program has reused the discarded labels C and D , by the way. Click **Edit|Highlights|Fill**, type ABE into the “polygon” box, click the yellow cell in the fill-color window, and click **fill**. The large triangle now has a yellow interior. Type CDF into the edit box, click the red cell, and click **fill** again. Notice that it was necessary to color the large triangle *first*. Close the dialog box. The figure should look like the illustration.

$$CDF/ABE = 0.28$$



46. Open the **Meas** dialog box, ask for the ratio of areas CDF/ABE (remember that it is all right to use lower case), and close the dialog. The display should show that the area of triangle CDF is 28% of the area of triangle ABE .

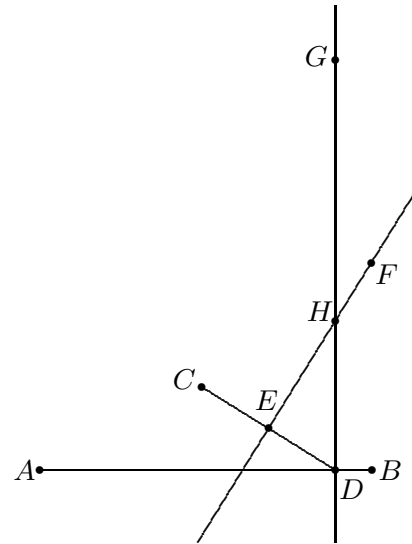
47. To change the number of decimal places in displayed calculations, open the dialog box **Edit|Decimals**, type a nonnegative integer less than 16, and press *Enter*. The new format will become apparent as soon as any numerical display is refreshed, which will happen when you change the value of $\#$ in the next paragraph.

48. Predict the value of the area ratio when C , D , and F are the midpoints of their respective sides. Set $\#$ equal to 0.5 to see whether you were right. Can the area of CDF be exactly 50% of the area of ABE ?

49. Make the drawing window the active window (by clicking its title, for example), press $Ctrl+L$ to turn off the labels, press $Ctrl+D$ a couple of times to turn off the vertex markers, then click **autoreverse** in the parameter dialog. Remember that you have to press the Q key to stop the animation.

Parabola Lab

0. Click the *Winggeom* menu item **Window|2-dim** to create a small drawing window.
1. To construct a parabola, you need a *directrix* and a *focus*. Put the mouse into **Btns|Segments** mode and right-click three points onto the drawing surface, with A near the bottom on the left, B near the bottom on the right, and C near the center of the window. Exact placement is not important, for adjustments will be made later. Now use the left button to connect A to B : Point at either vertex, hold down the left button, drag the pointer to the other vertex, and release. Segment AB appears.
2. Right-click a random point D onto segment AB . To check that D really is on the segment, put the mouse into **Btns|Drag vertices** mode and try to drag D . It should only slide along AB .
3. Put the mouse into **Btns|Segments** mode and use the left button to connect C to D .
4. To perpendicularly bisect segment CD , click **Line|Perpendiculars|Bisector**, type CD into the edit box, and press *Enter* (or click **ok**). The program first marks E at the midpoint of CD , then draws the perpendicular line EF . For inventory purposes, the program needs to maintain at least two points on every line, so it introduces F as well as E . All the points on line EF have a special property — what is it?
5. Click **Line|Perpendiculars|General** to draw the line that is perpendicular at D to segment AB . Type AB into the “perpendicular to” box (it might already be there) and D into the “through point” box. Click **draw** to see the line DG . The view will probably be adjusted because of these new points. Press *Escape* to close the dialog box.
6. The simplest way to label the intersection H of lines DG and EF is to point at it and click the right button. (If the intersection does not lie within the window, this method will not work, however. Another way is to click **Point|Intersection|Line-line**, type DG and EF into the two edit boxes, and press *Enter*.) The figure should now resemble the illustration.
7. Put the mouse into **Btns|Drag vertices** mode. Then use the left button to drag D back and forth along segment AB . If the menu item **Other|Autoextend** does not have a check mark, give it one by clicking it. This feature enables you to slide D past the ends of segment AB .
8. No matter what position D has on line AB , you can be certain that D is the point on AB that is _____ to H . No matter what position D has on line AB , what can you say about the distances from H to the focal point C and from H to the directrix AB ?



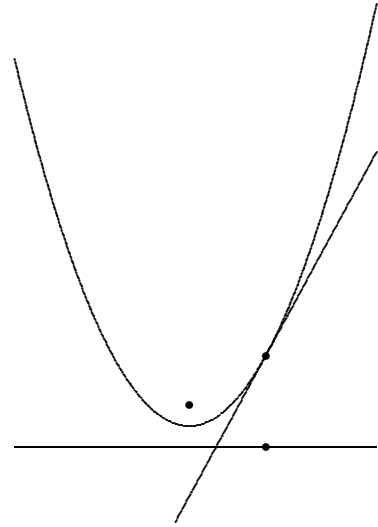
Parabola Lab

9. Now that point H has been constructed, points F and G are no longer needed, so click **Edit|Delete|Point**, type FG into the edit box, and press *Enter*. Lines DH and CD have also served their purposes, so click **Edit|Delete|Line**, type the list DH, CD into the edit box, and press *Enter*. Click **View|Window** (or press $Ctrl+W$) to reposition the figure in the window.

10. As you slide D along AB , try to visualize the path that H is following. Here is an easy way of making this path appear: Click **Animate|Temporary tracing**, type H into the edit box, and press *Enter*. When you slide D along AB now, the program retains all of the corresponding positions for H in the figure. It is better to move D slowly. This is only a temporary display (it disappears as soon as the screen is refreshed).

11. What would the path look like if C were closer to AB ? What would the path look like if C were further from AB ? To explore such questions, first drag C to a new position and then regenerate the path by sliding D along AB again.

12. For a smooth, permanent tracing, click **Animate|Tracing**, then click **new** in the dialog box. Click the **control vertex** button and type D into the adjacent edit box. This tells the program that the tracing is generated by dragging vertex D . You can leave the default values for “steps” (100), “low” (0.0), and “high” (1.0) as they are. Click **pen on vertex**, type H into the edit box, and press *Enter* (or click **ok**). The program plots H for 100 positions of D , and connects the dots. It is now necessary to close the tracing inventory dialog box.



13. Click **Animate|Temporary trace**, empty the list in the edit box, and press *Enter*. This disables the temporary trace path (and unchecks the menu item).

14. Press $Ctrl+L$ to turn off the labels. They are no longer needed. Press $Ctrl+D$ until all the vertices disappear. Open the dialog box **Edit|Labels|Individual**, to highlight specific vertices: Type CDH into the edit box, uncheck **show label**, and check **bullet**. When you now click **apply**, the three vertices C , D , and H should become visible, each identified by a solid circle, as in the diagram. Close the dialog box.

15. The perpendicular bisector EH has been left in the figure for a reason — it bears a special relationship to the parabola traced by H . As you slide D along AB , think of some words to describe this relationship: _____.

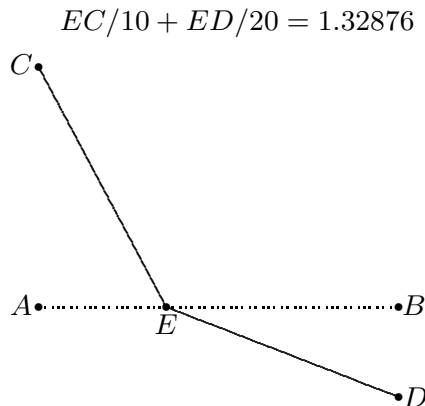
16. If you want the tracing to respond immediately when the vertices of the diagram are moved (C , for example), check the menu item **Animate|Monitor tracings**. If this item is not checked, it is necessary to click **Animate|Retrace** to redraw tracings.

Snell's Law Lab

0. Click the *Winggeom* menu item **Window|2-dim** to create a small drawing window.

1. Click **Point|Coordinates**. In the dialog box, type 0 into the x and y edit boxes. Press *Enter* (or click mark) to put $A = (0,0)$ onto the screen. Type 12 into the “x” box and press *Enter* again to make $B = (12,0)$ appear. In the same way, mark points $C = (0,8)$ and $D = (12,-3)$. Press *Escape* to close the dialog box. Click **View|Window** (or press *Ctrl+W*) to center the figure within the window. Press *Ctrl+A* to remove the axes.

2. Check that the mouse is in **Btns|Segments** mode, then use the left button to draw the segment AB . Use the right button to click a point E onto segment AB . Then use the left button to draw segments EC and ED .



3. Uncheck **Other|Measurement|Units**. Open the **Meas** dialog box and put the sum $EC/10 + ED/20$ on display. Press *Escape* to close the dialog box. The figure represents an interface AB between two media, and CED is a path followed by an object whose speed is 10 in the upper medium and 20 in the lower medium. The displayed measurement is the total time needed to traverse the path.

4. With the mouse in **Btns|Drag vertices** mode, slide E back and forth along segment AB , trying to make the displayed measurement as small as possible.

5. Once E is in its optimal position, open the **Meas** dialog, ask for the length of AE , and press *Escape* to close the dialog box. Before moving on, record the two values displayed in the figure: $AE = \underline{\hspace{2cm}}$ and $EC/10 + ED/20 = \underline{\hspace{2cm}}$.

6. If the objective had been to make the value of the sum $EC/10 + ED/10$ as small as possible, your search would not have led you to the same point E . Guess which point on segment AB would have been the optimal choice for E , then check your guess by doing the search. Begin by using the **Meas** dialog to delete $EC/10 + ED/20$ and replace it by $EC/10 + ED/10$. Finish this example by recording your findings: $AE = \underline{\hspace{2cm}}$ and $EC/10 + ED/10 = \underline{\hspace{2cm}}$.

7. New example: Search for the point E that makes $EC/20 + ED/10$ as small as possible. Record your findings: $AE = \underline{\hspace{2cm}}$ and $EC/20 + ED/10 = \underline{\hspace{2cm}}$. Notice that segment EC is longer than it was in either of the two preceding solutions. Explain why this could have been predicted.

8. Click **Line|Segments**, type the list AC, BD into the edit box, and press *Enter*. Segments AC and BD should appear. Use the mouse to drag E along segment AB . As E moves from A to B , what happens to the sizes of angles ACE and BDE ? Are they ever the same size?

Snell's Law Lab

9. You have now explored three problems of the same general form: Find the point E that minimizes the value of the expression $EC/r_C + ED/r_D$. The expression represents *time*, because the numerators of the fractions represent *distances* and the denominators represent *rates*. A few more examples and measurements will help to reveal a pattern. Fill in the missing entries of the table below, and notice that there are two new quantities of interest. Remember that each row requires a separate search.

r_C	r_D	EA	$EC/r_C + ED/r_D$	EA/EC	EB/ED
10	30				
10	20				
10	10				
40	30				
20	10				
30	10				

10. As the rate r_D decreases, what happens to the optimal position of the point E ? What happens to the length of segment ED ? What happens to the size of angle BDE ? Is it correct to say that r_D and angle BDE linearly related?

11. The simple pattern known as Snell's Law relates r_C , r_D , EA/EC , and EB/ED . It can be inferred by examining the table. Express this relationship in words. Express it in symbols.

12. Write a trigonometric description of Snell's Law that relates the rates r_C and r_D to the angles ACE and BDE .

Ellipse Lab I

0. Click the *Winggeom* menu item **Window|2-dim** to create a small drawing window.

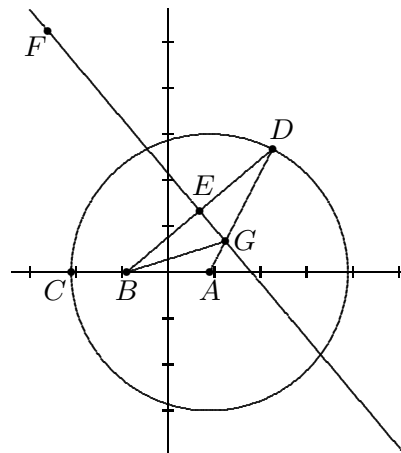
1. Click **Point|Coordinates**. Type 9 into the x box and 0 into the y box, and press *Enter* (or click **mark**). Point $A = (9, 0)$ appears on the screen. In the same way, mark points $B = (-9, 0)$ and $C = (-21, 0)$. Press *Escape* to close the dialog box.

2. Click **Circle|Radius-center**, type A into the “centered at” box, click **circle through**, type C into the adjacent box, and click **draw**. You should now see the circle centered at A that goes through C . Press *Escape* to close the dialog box.

3. With the mouse in **Btns|Segments** mode, point anywhere on the circle and right-click to mark D . Use the left button to draw segments AD and BD .

4. The point D is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns|Drag vertices** mode, point at D , press the left button, and hold it down while you slide the mouse.

5. Open the **Line|Perpendiculars|Bisector** dialog, type BD into the edit box, and press *Enter*. Notice that the program marked midpoint E on segment BD when it drew the bisector EF .



6. Put the mouse into **Btns|Segments** mode. Right-click the point where line EF intersects segment AD . The intersection point should now be labeled G .

7. Use the left button to connect G to B . The figure should now resemble the illustration.

8. As you answer the following questions, you may wish to move D around the circle (first put the mouse back into **Btns|Drag vertices** mode) and observe what happens to the constructions, especially point G . The **Meas** dialog box may also be useful.

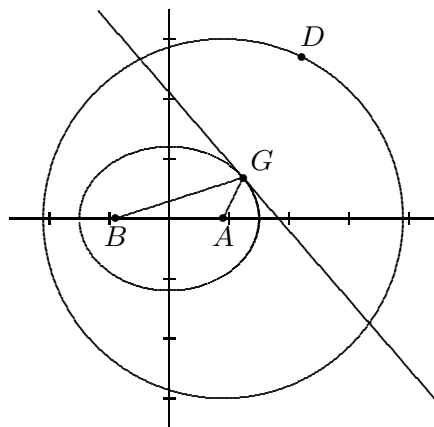
- (a) How do the lengths GB and GD compare?
- (b) What must always be true about the sum of the distances GB and GA , no matter what the position of point D ? Explain.
- (c) Describe what the trace of point G will look like as point D moves around the circle.

Ellipse Lab I

9. To see a picture of the path of G , open the inventory dialog box **Animate|Tracing** and click **new**. Click **control vertex** and type D into the adjacent box (this tells the program to produce the tracing by sliding D). Click **pen on vertex** and type G into the adjacent box. Use the default values for “steps” (100), “low” (0.0), and “high” (1.0). Press *Enter*. The program will plot 100 different positions for G , and connect the dots to form a curve. The construction is superimposed.

10. It is time to clean up a bit. Click **Edit|Delete|Line**, type DB, DA into the box, and press *Enter*. Click **Line|Segments**, type AG into the box, and press *Enter*. Click **Edit|Labels|Individual**, click **show label** to uncheck the box, and click **apply**. Then type $ABDG$ into the “vertices” box, check **bullet**, and click **apply**. Press *Escape* to close the dialog. Your figure should now resemble the illustration.

11. Click **Meas**, type G into the edit box, and press *Enter*. The coordinates of point G are added to the figure. Close this dialog box (press *Escape*) and return to sliding D around the circle. This makes G slide along a curve known as an *ellipse*. The path of G intersects the coordinate axes at four lattice points. Find these coordinates exactly. Is it a coincidence that the distance between the x -intercepts is equal to the radius of the circle? Explain.



12. Summarize the roles played by point A , point D , radius AC , and line EF in the construction of this ellipse.

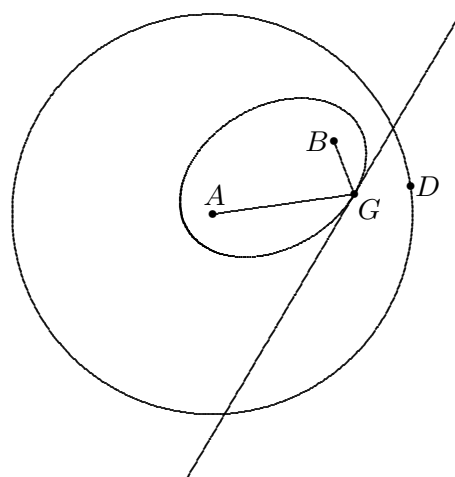
13. It so happens that the focal radius GA is always three fifths of the distance from G to the vertical line $x = 25$. Here is one way to confirm this fact: Use the **Point|Coordinates** dialog to plot two points on the line, say $H = (25, -30)$ and $I = (25, 30)$. Draw the line by clicking **Line|Lines**, typing HI into the edit box and pressing *Enter*. Then mark the point J on HI that is closest to G by clicking **Line|Perpendiculars|Altitude**, typing HI into the “perp to” box, G into the “from point” box, and pressing *Enter*. Now open the **Meas** dialog and ask for the values of GA , GJ , and GA/GJ , one at a time. Close this dialog and return to sliding D around the circle, keeping an eye on the displayed values. Notice that the ratio GA/GJ does indeed have a constant value, even though GA and GJ are not constant.

14. The line $x = 25$ is called a *directrix*. An ellipse has two of them. Can you find an equation for the other directrix of this ellipse?

15. The constant ratio GA/GJ ($3/5$ in this example) is called the *eccentricity* of the ellipse. Verify that the ratio AB/AD is also $3/5$. This is not a coincidence.

Ellipse Lab II

16. Click **File|New** to start a new sketch. Instead of plotting three specific points as in part I, put the mouse into **Btms|Circles** mode, and use the right button to mark three random points A , B , and C , with B closer to A than C is. Left-click A , drag the mouse to C , and release. This creates the circle in step 2 of part I. (If B is not inside the circle, drag it inside before proceeding.) Put the mouse into **Btms|Segments** mode and continue with the rest of the construction of G .



Unlike the specific points in part I, points A , B , and C can be dragged around the screen. It is interesting to see the effect that different positions have on the appearance of the elliptical path of point G .

17. Click **Animate|Tracing** and repeat step 9 of part I. For a more accurate tracing, put 300 into the “steps” box. Check the menu item **Animate|Monitor tracings**, so that this tracing will respond when you drag points other than D . **File|Save** the figure before proceeding. Drag B close to the center A . Notice that the shape of the ellipse is different this time. Now drag B close to the circle and notice the new shape. You can also experiment with changing the size of the circle, by dragging C . Notice that the construction falls apart if B is outside the circle. Keep in mind that the sum $GA + GB$ always equals the _____ of the circle. The sum $GA + GB$ is equal to what dimension of the ellipse itself? (see step 11 of part I)

18. The eccentricity AB/AD describes the shape of the ellipse. In the part I example, the eccentricity was $3/5$. Use the **Meas** dialog to display the value of AB/AD . When you move points A , B , or C , the eccentricity will change. What is the range of values of the eccentricity? What words would you use to describe the shape of the ellipse when the eccentricity has (a) a value near zero? (b) a value near one?

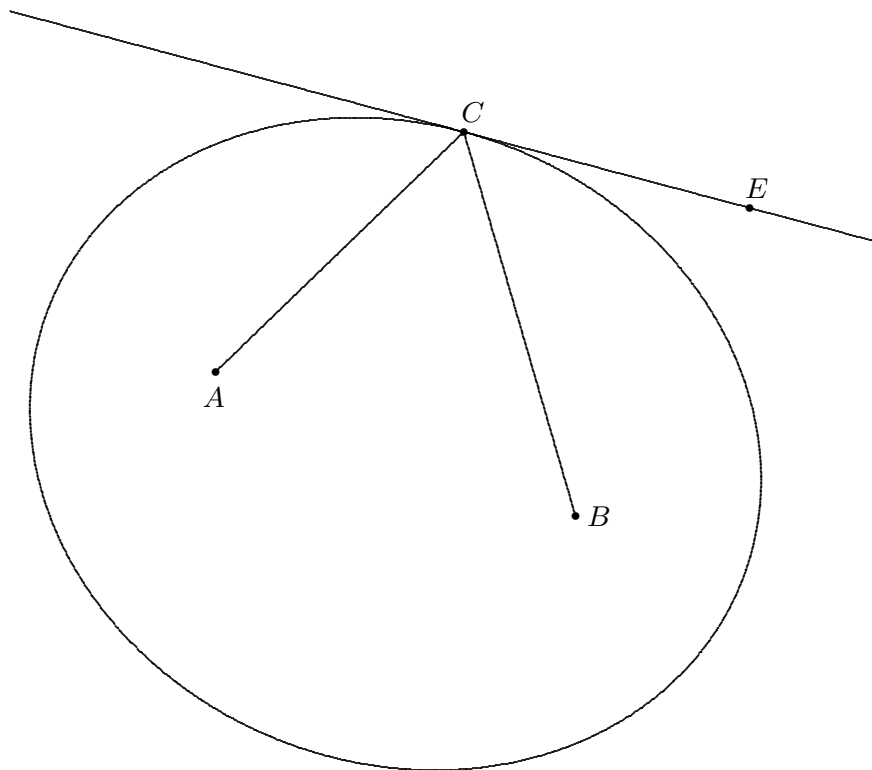
19. A Challenge: In part I, the lines $x = 25$ and $x = -25$ were the directrices for the ellipse. Recall that, for any point G on that ellipse, the distance from G to the focus A was three fifths of the distance from G to the corresponding directrix. This should help you construct the directrices for the generic ellipse in your figure. These lines should of course change whenever you change the ellipse by moving A , B , or C .

Ellipse Lab IV

23. Click **File|New** to make a fresh start. Put the mouse in **Btms|Segments** mode, right-click three points A , B , and C onto the screen, and use the left button to draw segments AC and BC .

24. Suppose that points A and B represent the focal points of an ellipse, and that C is a point on that ellipse. The problem is to construct the line that is tangent to the ellipse at C . This can be done in just two steps. First click **Line|Angles|Bisect old**, type ACB into the edit box, and press *Enter*. The bisecting ray CD should appear. Next click **Line|Perpendiculars|General**, type CD into the “perpendicular to” box, type C into the “through point” box, and click **draw**. Tangent line CE appears. *Escape* from the dialog.

25. Because there is no longer a need for the bisecting ray, click **Edit|Delete|Line** to remove CD , and click **Edit|Delete|Point** to remove point D . Now put the mouse into **Btms|Drag vertices** mode. Drag C around the screen and notice how the line CE reacts. Because the sum of lengths $AC + BC$ is not being kept constant, each line CE is tangent to a different ellipse. To see the ellipses, open the **Units|3-Point conics** dialog box. Click **ellipse**, type AB into the “focal points” box, type C into the “point on conic” box, and click **draw**. The ellipse is added to the conic inventory, and the curve appears on the screen. *Escape* from the dialog. Watch what happens when you drag A , B , and C around the screen.

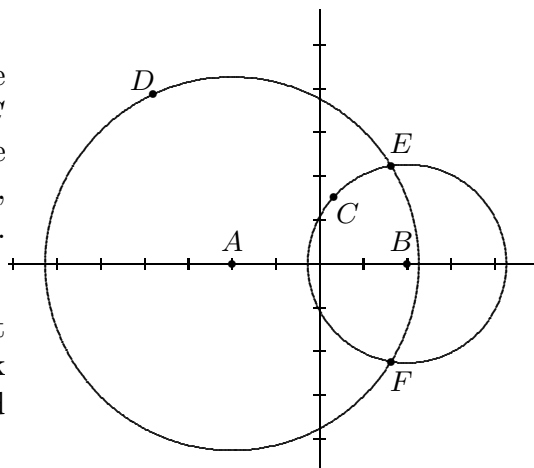


Hyperbola Lab I

0. Click the *Winggeom* menu item **Window|2-dim** to create a small drawing window.
1. Open the **Point|Coordinates** dialog box. Type -2 into the x box and 0 into the y box, then press *Enter* to make $A = (-2, 0)$ appear on the screen. In a similar fashion, make $B = (2, 0)$ appear. Press *Escape* to close the dialog box.

2. With the mouse in **Btns|Segments** mode, right-click a random point C onto the screen approximately midway between A and B .

3. Click **Circle|Radius-center**, type B into the “centered at” box, click **circle through**, type C into the adjacent box, and click **draw**. Now type A into the “centered at” box, click **circle radius**, type $2+BC$ into the adjacent box, and click **draw**. *Escape* from the dialog box.



4. Unless C is too close to B , the circles you just drew should intersect in two places. Right-click either point. The labels E and F will be applied to the intersections.

5. The coordinates of E and F can be displayed: Open the **Meas** dialog box, type E and press *Enter*, type F and press *Enter*, then press *Escape*.

6. Put the mouse into **Btns|Drag points** mode. Drag point C (which is actually the only point that can be dragged). You will see the coordinates for many points that are 2 units closer to B than they are to A . The y -coordinate of one of these intersection points is 5; its x -coordinate is approximately _____. Another special case occurs when the intersection points merge; the resulting x -coordinate is _____.

7. Click **Animate|Temporary trace**, type EF into the edit box, and press *Enter*. When you drag C around the screen now, the program accumulates all of the corresponding positions for E and F in the figure. More points are plotted when you move C slowly. This is only a temporary display, however — it disappears as soon as the screen is refreshed. A more permanent display is obtained next. The curve is called a *hyperbola*.

Hyperbola Lab II

8. Begin a fresh sketch with the same points A and B . One way to do this is to click **Edit|Undo** (or press $Ctrl+Z$) until C disappears. Another way is to make a fresh start with **File|New**, then use the **Point|Coordinates** dialog box to re-enter the coordinates for $A = (-2, 0)$ and $B = (2, 0)$.

9. Open the **Circle|Radius-center** dialog box. Type A into the “centered at” box, click the **circle radius** button, type 2 into the adjacent box, and click **draw**. A circle of radius 2 appears, with a random point C marked on it. Press *Escape* to close the dialog box.

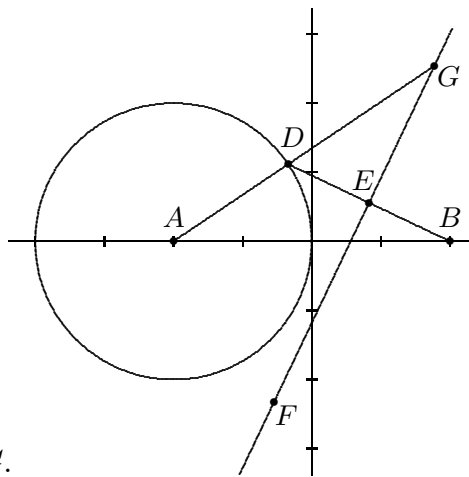
10. With the mouse in **Btns|Segments** mode, right-click a random point D onto the circle, fairly close to the y -axis. Use the left button to draw the segment BD . With the mouse in **Btns|Rays** mode, use the left button to draw the ray AD .

11. Click **Line|Perpendiculars|Bisector**, type BD into the edit box, and press *Enter*. Notice that the program automatically marks the midpoint E of segment BD when it draws the bisector.

12. Point at the intersection of ray AD and line FE , and right-click. You should see label G appear, as in the illustration.

13. Explain why $GA - GB = AD = 2$, no matter what the position of D .

14. Click **Animate|Temporary trace**, type G into the edit box, and press *Enter*. Put the mouse into **Btns|Drag vertices** mode and drag D slowly around the circle. Explain why the trace of G consists of points that already appeared in step 7 above. Notice also that, for many positions of D , there is no point G .



15. For a smooth, permanent tracing, open the **Animate|Tracing** dialog box and click **new**. Click **control vertex** and type D into the adjacent box (this tells the program to produce the tracing by sliding D). Click **pen on vertex** and type G into the adjacent box. Use the default values for “steps” (100), “low” (0.0), and “high” (1.0). Press *Enter*. The program will plot 100 different positions for G , and connect the dots to form a curve. The construction is superimposed.

Hyperbola Lab III

16. Click **File|New** to start a new sketch, and put the mouse into **Btns|Segments** mode. If the coordinate axes are still on the screen, press $Ctrl+A$ to remove them. Use the right button to mark two random points A and B , and a third random point C that is closer to A than B is.

17. Click **Circle|Radius-center**, type A into the “centered at” box, click **circle through**, type C into the adjacent box, and click **draw**. You should now see the circle centered at A that goes through C . Press *Escape* to close the dialog box.

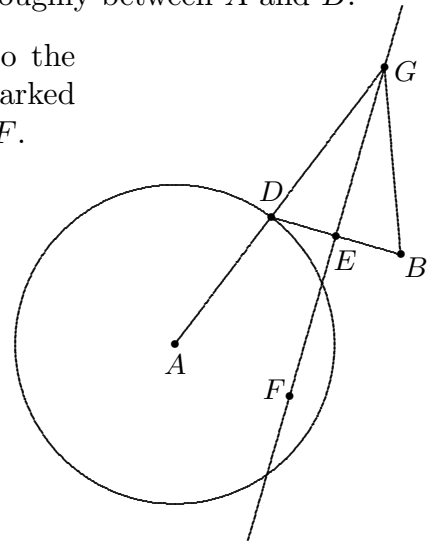
18. Point anywhere on the circle and click the right button to mark D . Then use the left button to draw the segments AD and BD .

19. The point D is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns|Drag vertices** mode, point at D , press the left button, and hold it down while you slide the mouse. When you are done, leave D roughly between A and B .

20. Click **Line|Perpendiculars|Bisector**, type BD into the edit box, and press *Enter*. Notice that the program marked midpoint E on segment BD when it drew the bisector EF .

21. So that the next label will appear, check the menu item **Other|Autoextend** by clicking it. Open the dialog **Point|Intersection|Line-line**, type EF into one box and AD into the other, and click mark. Label G should now mark the intersection of line EF and ray AD . Press *Escape* to close the dialog box.

22. Put the mouse into **Btns|Segments** mode, and use the left button to draw BG . The figure should now resemble the illustration.



23. As you answer the following questions, you may wish to move D around the circle (first put the mouse back into **Btns|Drag vertices** mode). Observe what happens to the constructions, especially point G . The **Meas** dialog box may also be useful.

(a) How do the lengths GB and GD compare?

(b) What must always be true about the difference $GA - GB$, no matter what the position of point D ? Respond carefully — this difference is not always positive.

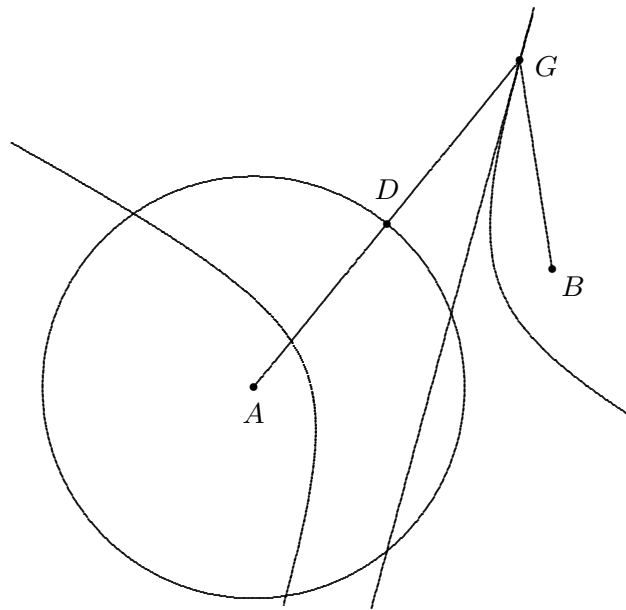
(c) Describe what the trace of point G will look like as point D moves around the circle.

Hyperbola Lab III

24. To see a picture of the path of G , open the inventory dialog box **Animate|Tracing** and click **new**. Click **control vertex** and type D into the adjacent box (this tells the program to produce the tracing by sliding D). Click **pen on vertex** and type G into the adjacent box. Use the default values for “steps” (100), “low” (0.0), and “high” (1.0). Press *Enter*. The program will plot 100 different positions for G , and connect the dots to form a curve called a *hyperbola*. The construction is superimposed.

25. It is time to clean up a bit. Click **Edit|Delete|Line**, type DB into the box, and press *Enter*. Click **Edit|Labels|Individual**, click **show label** to uncheck it, type EF into the “vertices” box, and click **apply**. Press *Escape* to close this dialog. Your figure should now resemble the illustration.

26. With the mouse in **Btms|Drag points** mode, you can drag A , B , and C around the screen and see the effect that this has on the tracing. To make this tracing respond when you drag points other than D , check the menu item **Animate|Monitor tracings**. Before proceeding further, **File|Save** the figure. It is interesting to see what happens to the trace when B is closer to A than C is — the hyperbola turns into an ellipse!



27. Summarize the roles played by point A , point D , radius AC , and line EF in the construction of this hyperbola.

Hyperbola Lab IV

28. Click **File|New** to make a fresh start. Put the mouse in **Btms|Segments** mode, right-click three points A , B , and C onto the screen, and use the left button to draw segments AC and BC .

29. Suppose that points A and B represent the focal points of a hyperbola, and that C is a point on that hyperbola. The problem is to construct the line that is tangent to the hyperbola at C . This can be done very quickly. First click **Line|Angles|Bisect old**, type ACB into the edit box, and press *Enter*. Bisecting ray CD should appear. To make it into a tangent line, click **Line|Extensions**, type DC into the edit box, and press *Enter*.

30. Now put the mouse into **Btms|Drag vertices** mode. Drag C around the screen and notice how the line CD reacts. Because the difference of lengths $AC - BC$ is not being kept constant, each line CD is tangent to a different hyperbola. To see the hyperbolas, open the **Units|3-Point conics** dialog box. Click **hyperbola**, type AB into the “focal points” box, type C into the “point on conic” box, and click **draw**. The hyperbola is added to the conic inventory, and the curve appears on the screen. *Escape* from the dialog. Notice what happens when you drag A , B , and C around the screen.

