Intersecting Lines Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

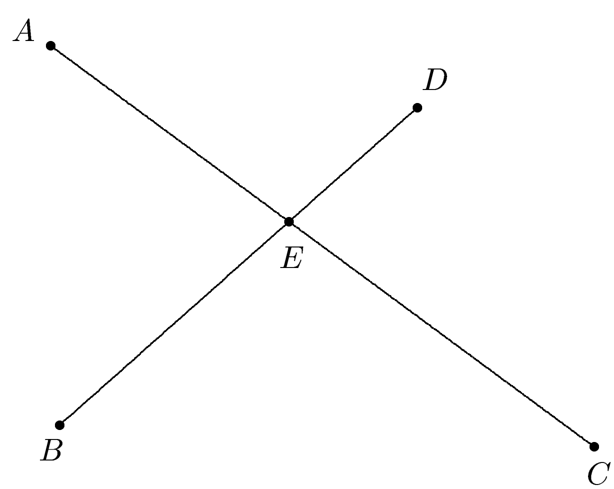
1. With the mouse in **Btns/Segment** mode, use the right button to create four points A, B, C, and D. Then use the left button to drag out the connections $\overline{AC}$ and $\overline{BD}$.

2. If these segments do not intersect, put the mouse into **Btns/Drag points** mode, then use the left button to drag one of the points until they do. Your figure might look somewhat like the illustration below.

3. Put the mouse back into **Btns/Segment** mode, point at the intersection, and click the right button. Label E should appear at the intersection.

4. Open the **Measure** dialog and type the following four items into the edit box, one at a time, pressing *Enter* after each: $\angle AEB$, $\angle BEC$, $\angle AEB + \angle BEC$, and $\angle CED$.

5. Put the mouse into **Btns/Drag points** mode, then use the left button to drag the vertices A, B, C, or D around the screen, noticing what happens to the displayed measurements as you do. The measurement $\angle DEA$ was omitted from the list. Explain how to predict what its value would be.



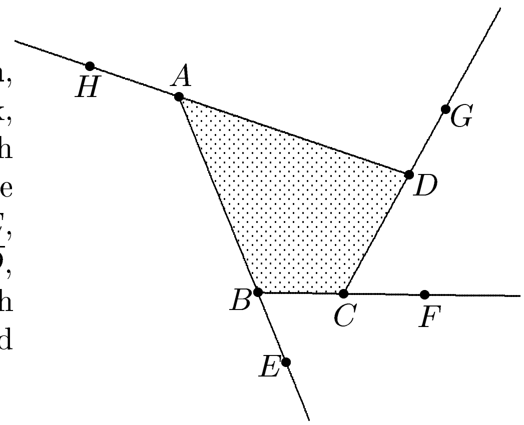
Sentry Theorem Lab

The subject of this lab is a theorem that simulates the experience of a sentry who has been given the job of walking around a polygonal building. Walk, turn, walk, turn, ...

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. Click **Shape/Random/Convex**, type 4 into the edit box, and press *Enter*. You should see a random quadrilateral $ABCD$ appear.

2. Extend the fours sides: Click **Line/Extension**, type the list AB, BC, CD, DA into the edit box, and press *Enter*. Notice that the order in which you type the letters is important. With the mouse in **Btns/Segment** mode, right-click the points E, F, G, and H onto the extensions of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Compare your figure with the illustration at right, which has been labelled in a consistent, counterclockwise fashion.



3. Open the **Measurement** dialog and type (pressing *Enter* after each) the items $\angle HAB$, $\angle EBC$, $\angle FCD$, $\angle GDA$, and $\angle HAB + \angle EBC + \angle FCD + \angle GDA$ into the edit box. The first four numbers should appear random, but the last one might surprise you. Could you have anticipated the sum? Think of the sentry. What is the meaning of an *exterior* angle such as HAB?

4. Put the mouse in **Btns/Drag points** mode and then use the left button to click and drag the vertices of $ABCD$ around the screen, noticing what happens to the displayed measurements.

5. It is possible to drag the vertices of $ABCD$ so that a non-convex quadrilateral is formed. Is the displayed data what you expected?

6. At this time, you may be ready to formulate a general theorem — one that applies to polygons other than just quadrilaterals. If necessary, apply the preceding steps (suitably modified) to a pentagon or a triangle.

7. Explain how the preceding can be used to calculate the angles (first exterior, then interior) for a *regular* polygon.

8. Now let's take a look at our figure from a great distance. Click **View/Expand frame** (or press *Ctrl+E*, which is easier to do). This has the effect of shrinking the figure slightly. Repeat many times, until the individual vertices are indistinguishable. What do you see? By the way, to restore a normal view, click **View/Window**.

Angle Bisector Lab I

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. With the mouse in **Btns/Segment** mode, use the right button to create three points A, B, and C. Then use the left button to draw the connections $\overline{AB}$ and $\overline{BC}$.

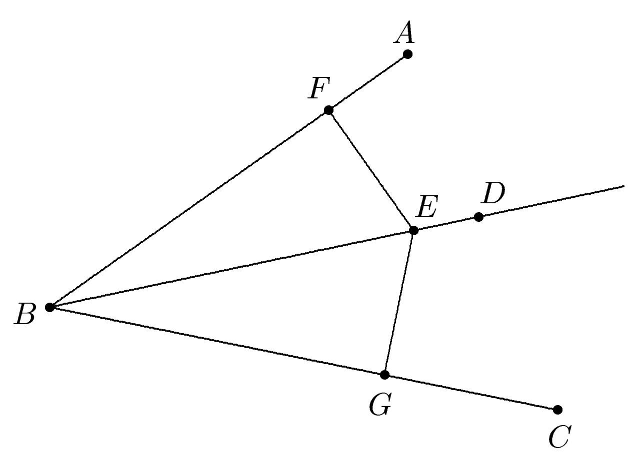
2. Click **Line/Angles/Bisect old**, type ABC into the edit box, and press *Enter*. The bisector $\overline{BD}$ should appear.

3. With the mouse in **Btns/Segment** mode, right-click a new point E onto $\overline{BD}$.

4. Click **Line/Perpendiculars/Altitude**. Type BA into the *segment* box, E into the *from point* box, and press *Enter* (or click **Mark**). You should see $\overline{EF}$, which meets $\overline{BA}$ perpendicularly at F. Now type BC into the *segment* box and press *Enter*, making $\overline{EG}$ appear, which meets $\overline{BC}$ perpendicularly at G.

5. Open the **Measure** dialog and type EF and EG into the edit box, pressing *Enter* after each. Could you have predicted the coincidence? What can be said about triangles BEF and BEG? What justification can you offer?

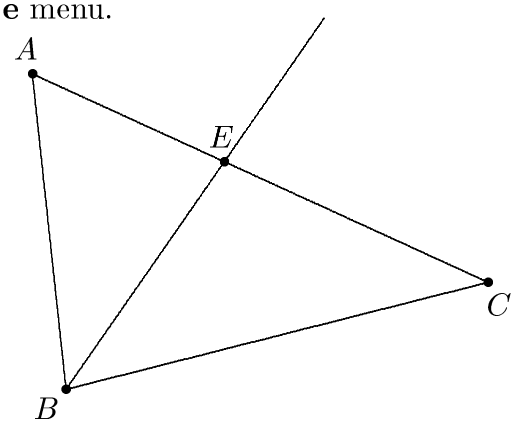
6. Put the mouse into **Btns/Drag points**, and use it to slide point E along $\overline{BD}$. Notice what happens to the displayed measurements as you do so. What property do all the points on an angle bisector have in common?



Angle Bisector Lab II

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. Obtain a random triangle ABC from the **Shape** menu.



2. Use the **Line** menu to bisect angle ABC.

3. With the mouse in **Btns/Segment** mode, use the right button to label the point E where the bisector $\overline{BD}$ intersects $\overline{AC}$.

4. Use the **Edit** menu to delete point D.

5. Click **Measure**. Ask for the measurements BA, BC, and AE/EC (pressing *Enter* after typing each), then close the dialog. One of the displayed measurements should reveal that E is not the midpoint of $\overline{AC}$.

6. Put the mouse into **Drag points** mode, then use the left button to drag the vertices of ABC around the screen. Try first to make the ratio AE/EC equal 1. In order for this to happen, what must be true of BA and BC?

7. Now try to find an arrangement that makes $\overline{BA}$ twice as long as $\overline{BC}$. When you find one, notice the value of AE/EC. Hmmm.

8. Formulate a statement that could be called *the Angle Bisector Theorem*. This should describe the location of the point where any angle bisector meets the opposite side of a triangle. Make additional measurements to confirm your statement.

9. Use the theorem to calculate AE, given that $AB = 9$, $BC = 15$, and $CA = 12$.

10. Numerical evidence neither proves nor explains. One way to understand where the angle-bisector proportion comes from is to make an additional construction: Use the **Line** menu to extend $\overline{CB}$, and to draw a line parallel to $\overline{BE}$ through A. Because the intersection of this line with $\overline{BC}$ might not be visible, use the **Point/Intersection/Line-line** dialog to label this point (it should be F, unless you have altered the previous steps). Then press *Ctrl+W* to make everything visible.

11. **Measure** $\angle BAF$ and $\angle BFA$, and explain why they should be equal. Measure BA and BF, and explain why they should be equal. Measure BF/BC and EA/EC, and explain why they should be equal. The conclusion of the theorem is now at hand.

Reflection Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. With the mouse in **Segment** mode, right-click five points A, B, C, D, E onto the drawing window, arranged so that triangle CDE is on one side of the line through A and B. To draw the line (a segment plus both extensions) through A and B, click **Line/Line**, type AB into the box, and press *Enter*. To see the triangle, you can click **Line/Segment**, or just use the left button to make the connections $\overline{CD}$, $\overline{DE}$, and $\overline{EC}$.

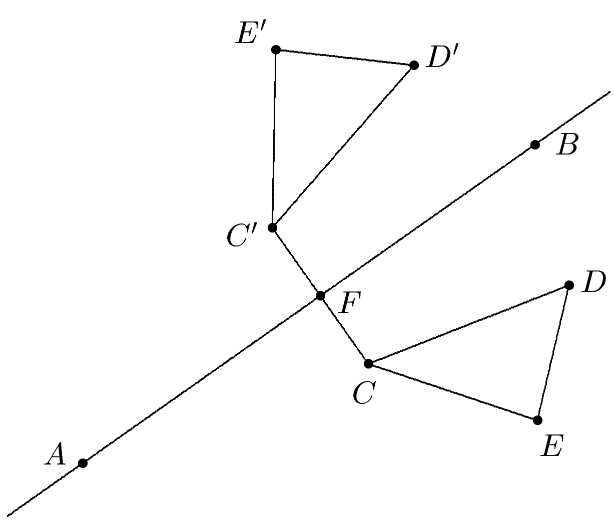
2. Click **Transform/Mirror**, check that AB appears in the *mirror* box, and type CDE into the *Apply to* box. Press *Enter* (or click **OK**). Image triangle $C'D'E'$ should appear.

3. Make the connection $\overline{CC'}$ with the left button, and right-click the label F onto the intersection of $\overline{CC'}$ and $\overline{AB}$.

4. Triangles CDE and $C'D'E'$ are of course congruent, but notice that their labellings have different orientations.

5. Before you make any confirming measurements, predict a relationship between $\overline{CC'}$ and $\overline{AB}$, between $\overline{DD'}$ and $\overline{AB}$, and between $\overline{EE'}$ and $\overline{AB}$. Now put the mouse into **Drag points** mode and use the left button to drag the vertices of triangle CDE around the screen. Notice what happens to the image triangle, and to the connector $\overline{CC'}$.

6. If you suspected that two congruent figures were related by a reflection, how could you confirm your guess and reconstruct the reflecting line (the mirror)?



Number Line Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. Put the mouse in its **BtnsSegment** mode, right-click two random points onto the screen, then use the left button to connect them with a segment. Click **View/Grid** to put the x- and y-axes on the screen.

2. Click **Point/Segment division**. (You can leave this dialog box on the screen for the rest of this session.) Type the coordinate 0.75 into the *Mark at* box, check that AB is displayed in the *segment* box, and press *Enter*. Notice where C is on $\overline{AB}$. Could you have predicted its location?

3. Type 1.5 into the *Mark at* box and press *Enter*. Notice where D is on the line AB. Could you have predicted its location?

4. Click **Measure**. Ask for the measurements AB, AC, and AC/AB (pressing *Enter* after typing each), then close the dialog. Which of the three values could you have predicted?

5. Type the coordinate -0.8 into the *Mark at* box and press *Enter*. Notice where E is on the line AB. Could you have predicted its location?

6. Put the mouse into **Drag points** mode and use the left button to drag A and B around the screen. Notice what happens to C, D, and E when you do so. Is their behavior predictable? Notice that you can not drag points C, D, and E individually. Is this as it should be?

7. Type BA into the *segment* box, 0.75 into the *Mark at* box, and press *Enter*. Point F should appear. Is the result the same as in step 2 above? Could you have predicted the result?

8. Click **Edit/Undo** four times (or press *Alt+backspace* four times) to remove C, D, E, and F. Then type AB into the *segment* box, $4/AB$ into the *Mark at* box, and press *Enter*. Notice the displayed measurements. Which one of them had a predictable value?

9. Use the left button to drag A and B around the screen. Notice what happens to C and to the displayed value of AC when you do so. Are the results predictable?

Parallel Lines Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. With the mouse in **Btns/Segment** mode, right-click two points A and B onto the blank screen, then use the left button to make the connection $\overline{AB}$. Then right-click points C and D onto $\overline{AB}$, and E somewhere off the line. Connect $\overline{CE}$.

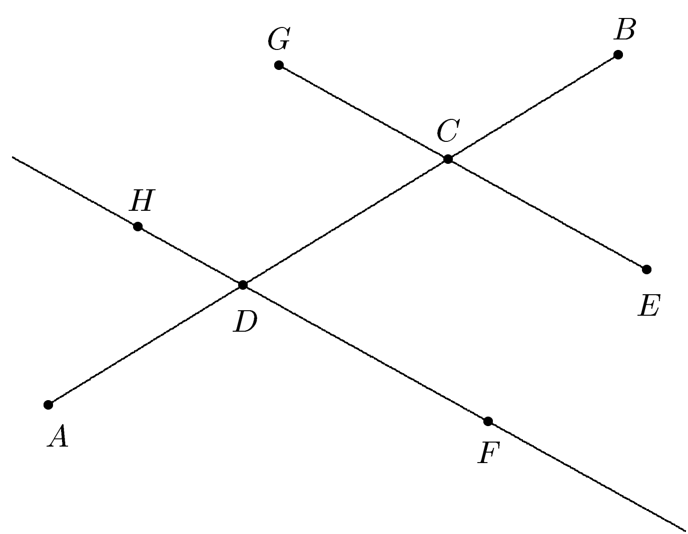
2. Click **Line/Parallels**, type D into the *through pt* box and CE into the *parallel to* box, and press *Enter*. You should see $\overline{DF}$ appear, parallel to $\overline{CE}$. Right-click point G onto $\overline{CE}$ and point H onto $\overline{DF}$.

3. Check **Other/Autoextend** by clicking it. This enables points to appear on the extensions of segments, necessary for the next request. Put the mouse into **Btns/Drag points** mode and use the left button to slide G so that it is not on the same side of $\overline{AB}$ as E. Repeat the process with H and F. Notice what happens when you slide C and D along $\overline{AB}$, or what happens when you drag E around the screen.

4. Open the **Measurement** dialog, type $\angle BCE$, and press *Enter*. Find two more angles in the figure that seem to have the same size, and use the dialog box to confirm your guess.

5. Ask for the measure of $\angle CDH$. How does this number relate to the size of $\angle BCE$? Close the dialog box. Do your predictions continue to hold true when the figure is altered by sliding points around?

6. Summarize your findings concerning the eight angles formed when a *transversal* crosses a pair of parallel lines.



Pythagorean Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. To get started, click **Shape/Random/Right triangle**. A random right triangle should appear on the screen.

2. Click **Shape/Polygon/Attach**, make sure that 4 is showing in the *sides* box, type the list BA, AC, CB into the *to edges* box, and press *Enter*. Squares $BADE$, $ACFG$, and $CBHI$ should now be attached to the *outside* of triangle ABC. Because the program has been taught to label figures in a *counterclockwise* direction, you have to be careful not to confuse BA with AB when you type.

3. Click **Measurement**. Type $BADE$ and press *Enter* to see the area of square $BADE$. Type $ACFG + CBHI$ and press *Enter* to see the sum of the areas of squares $ACFG$ and $CBHI$. Notice the (expected) equality. Close this dialog box.

4. Click **Edit/Randomize**. A different random right triangle appears on the screen, with squares attached. Notice (as expected) that the measurements are still equal. This randomization step can be repeated until you get tired of doing it.

5. To make a fresh start, click **Edit/Delete all** (click *No* if the program asks you to save the figure). Then click **Shape/Random/Triangle** (notice: not a right triangle). A random triangle should appear on the screen. Redo steps 2 and 3. The two measurements probably do not agree. Return to the **measurement** dialog and ask for the size of angle BCA. (Remember to type $< BCA$.) Close the dialog box.

6. To enable the mouse to drag individual vertices, check the item **Btns/Drag points**.

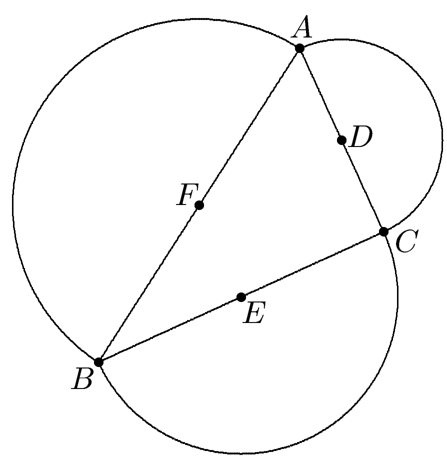
7. Point at any of the vertices of triangle ABC, hold down the left button, and drag the selected vertex to new positions. Notice that the pointer disappears and the vertex label changes color, to signal that you have engaged the vertex. Dragging vertices around is a way of randomizing the diagram. Notice that the displayed measurements change when the diagram does. Try to manuever triangle ABC so that the displayed values of $BADE$ and $ACFG + CBHI$ are equal, and watch what happens to the size of angle BCA.

8. By the way, if you try to drag one of the vertices other than A, B, or C, you will notice that the whole figure slides as a unit. Why do you suppose that this must happen?

9. Now for a surprise: Make a fresh start, redoing steps 1, 2, and 3, but instead of attaching squares to the outside of triangle ABC, attach *regular pentagons*. To do this, put 5 into the *sides* box in step 2. In step 3, you obtain the areas of the pentagons by typing five-letter descriptions — for instance, $BADEF$. Do you notice anything familiar about the measurements?

10. The previous item can of course be repeated with other polygons. Here is a slightly different exploration, however. Begin with a fresh random right triangle, and mark the midpoints of its sides: Click **Point/Segment division**. Type the list AC, CB, BA into the *segments* box, check that 0.5 is showing in the *coordinate* box, and press *Enter*. Close the dialog box.

11. Click **Circle/Radius-center** to draw some semicircles. These must be drawn one at a time. Type D into the *center* box, 180 into the *arc degree* box, DC into the *radius* box, and press *Enter*. A semicircle should now be attached to the outside of triangle ABC. (Notice that the program has relabeled A as G. It is possible to avoid this, by describing the arc size as $< CDA$ instead of as 180, but the simplest thing to do is just delete point G.) To see the other two arcs, use E and F as the centers and EB and FA as the radii, respectively. Notice that arcs are drawn in a counterclockwise direction, so that it makes a difference which end of the arc is designated as the initial vertex.



12. At any time, you can undo your last step, by clicking **Edit/Undo**.

13. To measure the areas of the semicircles, you must use the usual formula $\frac{1}{2}\pi r^2$. Click **Measure** to open the dialog, then type 0.5[pi]DC^2 and press *Enter* to see the area of the first semicircle. (The square brackets are necessary, for they tell the program not to look for a segment named PI. Or, you can press the F1 key instead of typing [pi].) Now type formulas for the areas of the other two semicircles. Are the answers related as you expected?

14. The Pythagorean Theorem is sometimes stated, *The square on the hypotenuse of a right triangle is equal to the sum of the squares on the legs.* Propose a more general version of this theorem, which takes into account the examples you have just seen.

Circumcenter Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

Part I

1. To get started, create a random triangle by clicking **Shape/Random/Triangle**.

2. Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$: The simplest way to proceed is to click **Line/Perpendiculars/Bisectors**, type the list AB, BC into the box, and press *Enter*. The screen will adjust to show $\overline{DE}$ and $\overline{FG}$, where D and F are midpoints.

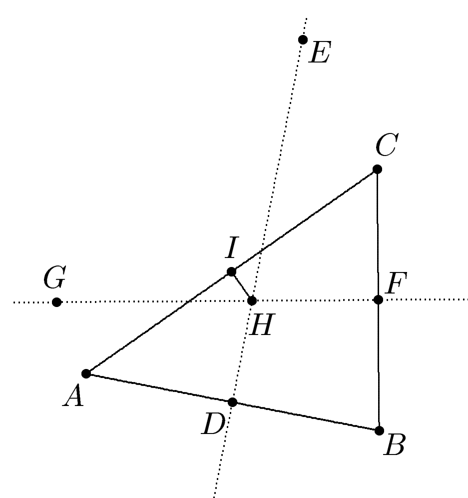
3. Label the intersection of the bisectors. There are two ways to do this. If the mouse is in **Btns/Segment** mode, just point at the intersection and click the right button. Label H should appear. (The other way to label an intersection point is to use the **Point/Intersection/Line-line** dialog. This might be preferred if it were difficult to see the intersection clearly.)

4. Mark the *midpoint* of $\overline{CA}$: Click **Point/Segment division**, type CA into the *segment* box, notice that the coordinate 0.5 already is showing in the *Mark at* box, and press *Enter*. Label I should appear at the midpoint.

5. Connect H and I. (There are two ways to do this.)

6. Open the **Measurement** dialog box, then ask for HC, HA, and $\angle HIA$, one at a time. Close the dialog box. Notice that the displayed measurements confirm (doubly) a theorem about the perpendicular bisectors of a triangle. What theorem?

7. Because ABC is a random triangle, its vertices can be dragged around the screen, while the rest of the figure and the measurements react. Put the mouse into **Btns/Drag points** mode, then use the left-button drag technique on A, B, and C to see that the displayed measurements continue to illustrate the theorem. (The other parts of the figure can not be dragged around — the whole figure moves instead.)



Circumcenter Lab

Part II

8. Click **Edit/Delete points**, type $EFGI$ into the box (commas are not needed for this list), and press *Enter*. This removes two of the lines as well as four of the points from the figure. What should remain is the triangle ABC and one of its perpendicular bisectors, namely DH, where H is the so-called *circumcenter* of ABC. You can delete the measurements from the drawing (one of them no longer makes sense).

9. Drag vertex C around the screen, noticing how H is constrained to move along the bisector $\overline{DH}$, which itself *does not move*. It is independent of C.

10. Drag vertices A and B around the screen, noticing that H is still forced to move along the bisector $\overline{DH}$, which must of course move because $\overline{AB}$ is moving.

11. Notice that the circumcenter H is sometimes inside the triangle, sometimes outside. This of course depends on the positions of the three vertices. Try to make sense of this inside-outside phenomenon. In particular, when the circumcenter drifts from inside to outside (or outside to inside), where does the crossing take place?

Part III

12. Make a fresh start, say by clicking **Edit/Delete all**. The program will ask you if you want to save your work. The three choices are

Yes: Save the figure (which you actually might have done already) and clear the screen;
No: Clear the screen without saving the figure;
Cancel: Forget that the delete request was ever made.

Choose either *Yes* or *No*, then request another **Shape/Random/Triangle**.

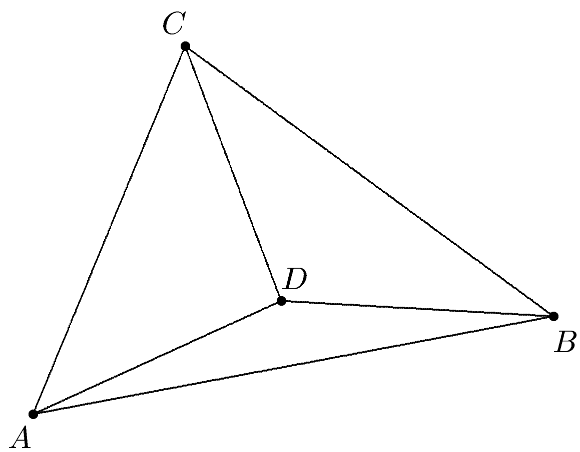
13. A quick way of making the circumcenter of ABC appear is to ask for the circumcircle. Click **Circle/Circumcircle**, notice that ABC already appears in the edit box, and press *Enter*. A circle and its center D should appear. Because we are not concerned with the circle today, we next delete it: Click **Edit/Delete circle**, type the center-radius description DA into the edit box (if it is not already there), and press *Enter*. The circle should disappear, leaving only its center D behind.

14. As in steps 9, 10, and 11, move the vertices of ABC around the screen, trying to make sense of the motion of the circumcenter D.

Part IV

15. Now we focus on the three isosceles triangles that have been in the background so far. Click **Line/Segments**, type the list DA, DB, DC into the box, and press *Enter*.

16. Click **Measure** and ask (one at a time) for $\angle DCA$, $\angle DCB$, and $\angle DBC$. Close the dialog box, and return to sliding the vertices of ABC around the screen. Notice the agreement of the second and third angles; why is it expected? Notice that the first and second usually do not agree. Is $\overline{CD}$ ever an angle bisector? Are any two of the three isosceles triangles ever congruent to each other?



17. Check the **Other/Autoextend** item by clicking it. Click **Point/Intersection/Line-line**, type AB and CD into the two boxes, and press *Enter*. Point E will appear on $\overline{AB}$, and $\overline{CD}$ will be extended to reach it, if necessary (this is the reason for activating the Autoextend feature). Slide the vertices of ABC around the screen. Is E ever the midpoint of AB? Is $\overline{CE}$ ever perpendicular to $\overline{AB}$? Make additional measurements if you need to.

18. For extra effect, you can color the triangles: Click **View/Highlight/Fill**, type DAB into the box, and click **Fill**. Repeat for triangles DBC and DCA, but click the **Color** button each time to change the color before you click **Fill**. To change your mind, use the **Empty** button. Press *Escape* to close the dialog. If the alphabetic labels get in the way of the colors, you can Press *Ctrl+L* to turn off the labels, or you can go into **Btns/Text** mode and drag the labels to new locations.

Midpoint Quadrilateral Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. To get started, click **Shape/Random/Convex**, type 4 into the edit box, and press *Enter*. A random convex quadrilateral $ABCD$ should appear on the screen.

2. Mark the midpoints of all four sides: Click **Point/Segment division**, type the list AB, BC, CD, DA into the *segment* box, notice that 0.5 already appears in the editing box, and click **Mark at**. The four midpoints E, F, G, and H should appear on $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Close the dialog box.

3. Connect the midpoints to form a new quadrilateral: Click **Line/Segments**, type $EFGHE$ into the edit box, and press *Enter*. (This dialog box accepts $EFGHE$ as an abbreviation for the list EF, FG, GH, HE. The same chaining shortcut would have worked in step 2.) Quadrilateral $EFGH$ should now be connected on the screen.

4. Because it was randomly chosen, quadrilateral $ABCD$ is not likely to have any special properties. The midpoint quadrilateral $EFGH$ is quite special, however. To see this, ask for the measurements EF, FG, GH, and HE. (The **Measurement** dialog only allows you to do these one at a time.) What kind of quadrilateral is $EFGH$?

5. If quadrilateral $ABCD$ *does* have special properties, then the midpoint quadrilateral $EFGH$ may have *additional* special properties. If you put the mouse into **Btns/Drag points** mode and drag the vertices of $ABCD$ around the screen, you can give $ABCD$ special properties and watch what happens to $EFGH$ as a result. Or, instead of this approach, you can *start* with a special $ABCD$ and construct its midpoint quadrilateral $EFGH$. For example, click **Shape/Random/Kite** to obtain a random kite, or **Shape/Random/Trapezoid** to obtain a random trapezoid. Whatever your approach, you may want to make additional measurements as you answer the following:

If $ABCD$ is a rectangle, then $EFGH$ is ______________________________ .

If $ABCD$ is a kite, then $EFGH$ is ______________________________ .

If $ABCD$ is a rhombus, then $EFGH$ is ______________________________ .

If $ABCD$ is an isosceles trapezoid, then $EFGH$ is ______________________________ .

If $ABCD$ is a parallelogram, then $EFGH$ is ______________________________ .

If $ABCD$ is a square, then $EFGH$ is ______________________________ .

If $ABCD$ is a nondescript quadrilateral, then $EFGH$ is ______________________________ .

6. Conversely, one can ask for those quadrilaterals $ABCD$ that produce specific properties in $EFGH$. Complete the following:

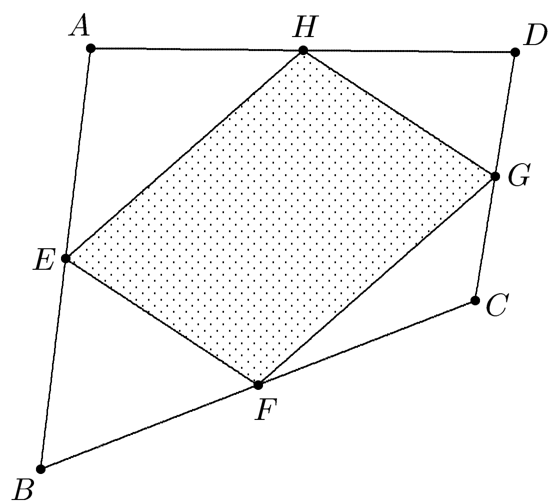
If $EFGH$ is to be a rectangle, then $ABCD$ must ____________________ .

If $EFGH$ is to be a rhombus, then $ABCD$ must ____________________ .

If $EFGH$ is to be a parallelogram, then $ABCD$ can ____________________ .

If $EFGH$ is to be a square, then $ABCD$ must ____________________ .

7. The (invisible) diagonals of $ABCD$ play a significant role in explaining the preceding patterns. What is their role?



8. A useful effect: Draw the diagonals $\overline{AC}$ and $\overline{BD}$ (use the left mouse button, or else click **Line/Segments**), and then highlight them. One way is to click **View/Highlight/Style line**, type AC into the edit box, click the *thickness 2* button, click **Redraw**, type BD into the edit box, and click **Redraw** again. Close this dialog box, then drag the vertices of $ABCD$ around and study the relationship between $EFGH$ and the diagonals $\overline{AC}$ and $\overline{BD}$.

9. A possible approach to the first question of item 6: Start with a random rectangle, and change its labels to $EFGH$ (first put the mouse into **Btns/Text** mode, then right-click individual labels to change them). Put the mouse back into **Bts/Segment** mode, and mark a random point A outside the rectangle, in the vicinity of E. Then build quadrilateral $ABCD$ so that $EFGH$ is its midpoint quadrilateral: First connect A to E, then use the **Point/Segment division** item to extend $\overline{AE}$ to B so that E is the midpoint of $\overline{AB}$. Next connect B to F, and so on. When you are done, notice that $ABCD$ has a special property, no matter where A is placed on the screen.

Mirror Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. To get started, click **Shape/Segment** and then **OK**. Horizontal $\overline{AB}$ should appear on the screen.

2. Draw the perpendicular bisector of $\overline{AB}$ by clicking **Line/Perpendiculars/Bisector**, typing AB into the edit box, and pressing *Enter*.

3. If this item is not already checked, click **Btns/Segments**, which enables the placing of new segments and points by using the mouse. Then place two new points on $\overline{CD}$, the second above the first, and both above point C, by pointing at them and clicking with the *right* button. The labels E and F should appear, the second above the first.

4. The preceding constructions model a floor ($\overline{AB}$) and a mirror ($\overline{EF}$) on a wall. The next thing to do is to clean up the extra lines: First turn off both extensions of $\overline{CF}$ (click **Line/Extension**, type the list CF, FC, and press *Enter*). Finally, delete point D (click **Edit/Delete pt**). Delete $\overline{CE}$ (click **Edit/Delete segment**).

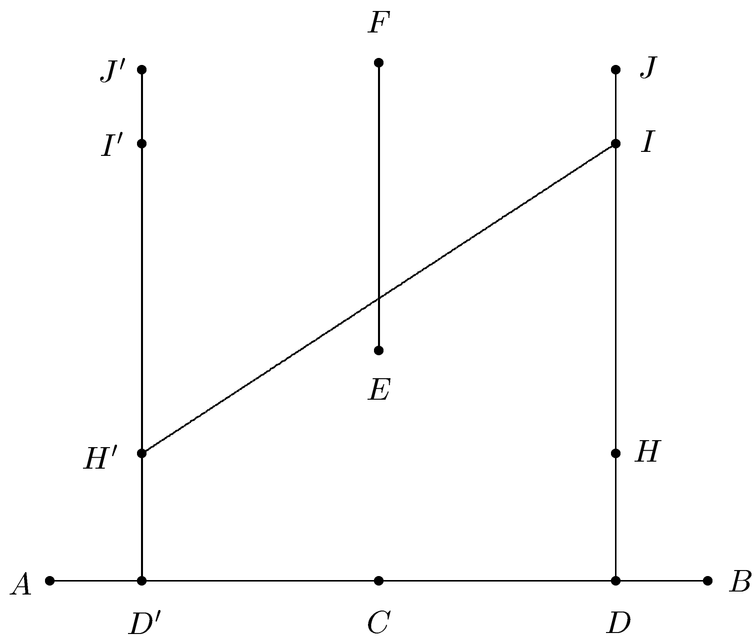
5. Now add a person to the figure: For the feet, click with the *right* button on a point on $\overline{CB}$. The label D should appear. For the body, click **Lines/Perpendiculars/General**, check that AB appears in the *segment* box, type D into the *perpendicular to* box, then press *Enter*. Line DG should appear. Working from bottom to top, right-click three points on this line, all above D. Think of them as representing the knees, the eyes, and the top of the head. The labels H, I, and J should appear. Finally, clean up the extra lines: Turn off the extensions of $\overline{DG}$ (click **Line/Extension**, type the list DG, GD, and press *Enter*). Delete point G (click **Edit/Delete pt**).

6. To create the mirror image of person $\overline{DJ}$, click **Transform/Mirror**. Type EF into the *mirror* box and $DHIJ$ into the *apply to* box. Click **OK** (or press *Enter*) and the mirror image $\overline{D'H'I'J'}$ should appear on the other side of the mirror.

7. Use the left mouse button to draw the sightline $\overline{IH'}$. To do this, hold down the left button on I, slide the pointer to H', then release. You should see $\overline{IH'}$. If this segment intersects mirror $\overline{EF}$, this means that H' (the image of H) is visible in the mirror. If not, the following item shows how to make adjustments in our model.

8. Put the mouse in point-dragging mode, by clicking **Btns/Drag point**. Then notice that points E, F, H, I, J, and D can all be slid along their respective segments. Point at one of them, hold down the left button, and slide the mouse. In this way, you can adjust the size of the mirror, where it is hung on the wall, the height of the person, the location of the eyes and knees, and where the person is standing.

If you try to drag other points, the entire figure will slide as a unit. Your drawing might now look something like this (in which H' is visible at I):



9. As when working with a word processor, it is generally a good idea to save your work as you proceed, to avoid having to redo all of the preceding steps in case of a crash. Click **File/Save**, choose a filename (at most eight characters) for this figure, type it into the edit box, and press *Enter*. From now on, you can quickly update the contents of this file by clicking **File/Save**. (To retrieve the file at any later time, click **File/Old**, find it in the directory, select it, and press *Enter*.)

10. The questions to be explored using this geometric model: How tall does a mirror have to be, to enable a person $\overline{DJ}$ to be able to see the entire image $\overline{D'J'}$, and where should the mirror be mounted on the wall? In particular, do the answers to these questions depend on the location of the person's I, or on the distance CD from the person to the mirror? You may want to use the **Measurement** dialog box to get numerical information about the person's height DJ, the size of the mirror EF, its distance CE above the floor, etc.

11. Think of some other interesting constructions or measurements to make.

Paper-Folding Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. To get started, click **Point/Grid**. The coordinates $x = 0.0$ and $y = 0.0$ should be showing in the edit boxes. Click **Label** to define $A = (0, 0)$. Then type 12 into the x box and click **Label** (or press *Enter*). Label B should appear at $(12, 0)$. (By the way, you can use the *Tab* key to move from one box to the other, or else click with the left button. Tabbing always highlights existing text, which means that it will be automatically replaced by whatever you type.) Type 12 into the y box and press *Enter* to see $C = (12, 12)$. Finally, type -36 into the x box and press *Enter* to see $D = (-36, 12)$. You can press *Escape* to close this dialog box, which removes the coordinate axes.

2. Click **Line/Segments**, type the list AB, BC, CD into the edit box (lower case letters are fine, by the way), and press *Enter*.

3. Click **Line/Extension**, type BA, CD into the edit box, and press *Enter*. The resulting two rays are supposed to complete the suggestion that we are working with a very long rectangular strip of paper.

4. Click **Point/Segment division**. We want to mark a movable point on segment AB. To do this, type AB into the *segment* box, type # (which stands for a variable quantity) into the *coord* box, then click **Mark at**. Point E should appear on AB.

5. We want to connect C to E. There are two ways this can be accomplished. If the mouse is in **Btns/Segment** mode, just click C with the *left* button, hold down your finger while you slide the pointer to E to draw the segment, then release. The other approach is to click **Line/Segments**, type CE into the edit box (lower-case letters are fine), and press *Enter* (or click **OK**). The segment should appear.

6. We want to mark the midpoint of the segment we just drew (so that we can then draw the perpendicular bisector) so return to the **Segment division** dialog box, make CE the segment, change the edit-box coordinate to 0.5, and click **Mark at**. Midpoint F should appear. Close this dialog box, which is no longer needed.

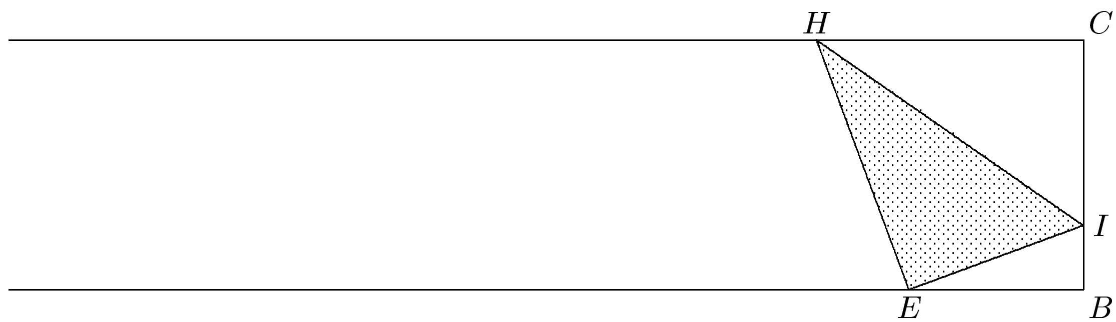
7. Click **Line/Perpendiculars/General**, type CE into the *perpendicular to* box, type F into the *through point* box, then press *Enter* (or click **Mark**). Line FG should appear. The view frame will be adjusted because of the new point G. Close this dialog box.

8. We want to label the points where the perpendicular bisector meets $\overline{CD}$ and $\overline{BC}$. There are two ways this can be done. One approach: Click **Point/Intersection/Line-line**. Type the segment names FG and CD into the boxes and press *Enter* (or click **Mark**). Label H should appear. Then replace CD with BC and press *Enter*. Label I should appear. Close the dialog box. (The other way to mark the intersections is to put the mouse into **Btns/Segment** mode, then click the intersection points with the *right* button. The labels H and I should appear.

9. Connect E to H and E to I. Use the easiest method.

10. The finishing touches:
Use the **Edit** menu to **delete points** F and G and to **delete segment** CE.
Click **Line/extension** to switch off the extensions of HI and IH.
Hide the labels A and D: Click **View/Labels/Individual**, type A, D into the edit box, uncheck the *Label* box, then click **Mark** (or press *Enter*). Close this dialog box.
Click **View/Window** to center the picture on the screen.
To heighten the folded-paper illusion: Click **View/Highlight/Fill**, type HIE into the edit box, then click **Fill**. (The default color/pattern is red/solid, which can be changed, of course.) Close this dialog box.

The figure should now look something like this:



11. Click **Measure**. Type BC into the box and press *Enter*. The information $BC = 12$ should appear (in two places). Type HI and press *Enter*, then type $< EHI$ and press *Enter*. Close the dialog box, and notice that the information is displayed with the figure.

12. Click **Animate/Parameter values**. This creates a scroll-bar control that allows you to vary the position of point E on $\overline{AB}$. The various positions are described by the single coordinate #, which has been set up so that 0 corresponds to A and 1 corresponds to B. You can change the position of E by typing a new value for # and pressing *Enter*, or by using the left mouse button to click and drag the scrolling button, or by clicking the left and right arrows. Try it. Notice that two of the displayed measurements change when the figure does. Do not click the two **reset** buttons, for they alter the $0 \leq \# \leq 1$ interval.

13. You can also put the scroll-bar in autopilot mode by clicking the **Autoreverse** button. To stop the animation, you will have to press a key, other than F (which increases the speed of the animation), S (which decreases the speed), or the spacebar (which reverses the direction). The scroll-bar control works better (for some reason, the screen refreshes faster) if it does not overlap the drawing window, so you may want to drag this little window out of the way.

14. Reopen the **Measurement** dialog box, highlight the $BC = 12$ item (by clicking it), then click the **Hide** button. This removes the item from the figure, where it is not really needed. Close the dialog box.

15. *Questions*: What pairs of segments in the figure are guaranteed to have the same lengths? What pairs of angles are guaranteed to have the same size? Verify your answers by making measurements. When you are done, you will probably want to **hide** (perhaps even **delete**) these measurements to avoid cluttering the figure.

16. *More questions*: What is the length of the crease $\overline{HI}$ when the fold angle CHI is 30°? What is the area of triangle BEI when angle CHI is 30°?

17. When using the program to search for numerical answers to questions like the preceding, the **reset** buttons below the parameter-value scroll bar are useful. Clicking the left **reset** button sets the leftmost position of the bar to the current parameter value, and clicking the right **reset** button sets the rightmost position of the bar to the current parameter value. If this is done to two values that are close together, the interval between these values can then be explored more accurately. (The scroll bar always divides the active interval into 100 pieces of equal width.) Try this technique on item 16.

18. *Some algebra*: Assign x to be the length of $\overline{CI}$. You should be able to label three other segments in the figure with their lengths. These labels allow you to answer item 16 without using the computer.

19. Using the same letter x, write a formula for the area of triangle BEI. You can then use your calculator to decide which of the many possible triangles BEI is the largest. This can also be done using the computer, too.

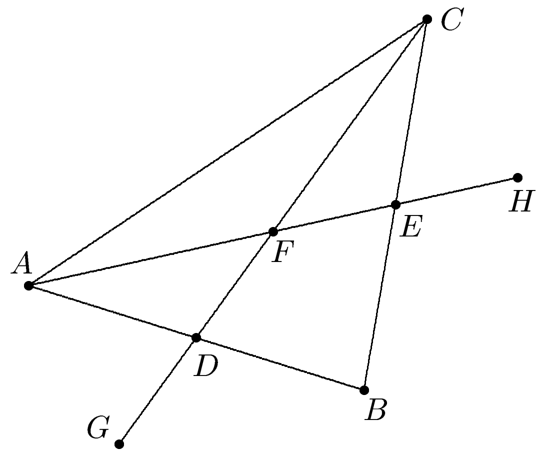
Centroid Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. Click **Shape/Random/Triangle** to obtain a random triangle.

2. Mark the midpoints D and E of $\overline{AB}$ and $\overline{BC}$: Click **Point/Segment division**, type the list AB, BC into the *segment box*, and press *Enter*.

3. With the mouse in **Btns/Segment** mode, use the left button to draw the medians $\overline{AE}$ and $\overline{CD}$. Then right-click the label F onto the intersection of $\overline{AE}$ and $\overline{CD}$.



4. Extend $\overline{FD}$ and $\overline{FE}$ their own lengths to points G and H, respectively. (In other words, we want $FD = DG$ and $FE = EH$.) To do this, click **Point/Segment division** (this dialog might still be open from step 2). Then type the list FD, FE into the *segment* box, the coordinate 2 into the edit box, and press *Enter*. The figure should now look something like the illustration at right.

5. Click **Measure**, type AF/FE and press *Enter*. A familiar number should appear. Our goal is to discover why, as well as to show that the three medians of any triangle are concurrent. Close the dialog box.

6. So that we do not lose sight of the triangle during the following construction, let us highlight it. Click **View/Highlight/Style line**. Type the list AB,BC,CA into the edit box, click the **thickness 2** button, and click **Apply** (or press *Enter*). Close the dialog.

7. Click **Line/Segment**, type $AGBF$ into the box, and press *Enter*. What kind of quadrilateral does $AGBF$ seem to be? Put the mouse into **Btns/Drag points** mode and use the left button to move the vertices of triangle ABC around the screen — does your answer change? Can you prove your response?

8. Connect $\overline{BH}$ and $\overline{HC}$. What kind of quadrilateral is $BHCF$? How do you know?

Centroid Lab

9. It should *look* like $\overline{FH}$ is parallel to $\overline{GB}$, and that $\overline{FG}$ is parallel to $\overline{HB}$. Give evidence to show that this is true. What kind of quadrilateral is $BHFG$?

10. Find two segments in your figure that you are certain have the same length as $\overline{AF}$, no matter how the vertices of ABC are placed on the screen.

11. Find two segments in your figure that you are certain have the same length as $\overline{CF}$, no matter how the vertices of ABC are placed on the screen.

12. How do the lengths of $\overline{AF}$ and $\overline{FE}$ compare?

13. How do the lengths of $\overline{CF}$ and $\overline{FD}$ compare?

14. What would happen if the median from B to $\overline{CA}$ were now drawn? *How do you know*?

15. Point F is called the *centroid* of triangle ABC. Summarize in a couple of sentences what you have learned about this point.

Color Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. (Items in boldface are meant to be clicked with the mouse.) This creates a small drawing window.

Part I: Color mixing

1. Click **Shape/Random/Triangle**, then right-click a random point D into the interior of your random triangle. Connect D to A, B, and C.

2. Put the mouse into **Btns/Text** mode, and use it to change the labels on the vertices. Right-click label A and you will be given the opportunity to replace it with a new label. Type R and press *Enter*. Then right-click label C, type G, and press *Enter*. The triangle should now be named RGB. Finally, change label D to C. It will be assumed from now on that you have made these changes.

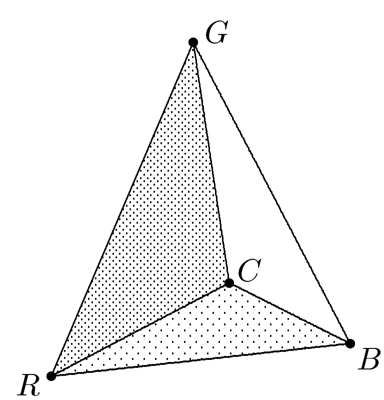
3. Click **View/Highlights/Fill**. Type CGB into the edit box box and press *Enter* (which is the same as clicking **Fill**). You should see a solid red triangle. Click the **Color** button once (which sets green as the next color to be applied), type CBR into the box, and press *Enter*. A solid green triangle appears. Finally, click the **Color** button once (which sets blue as the next color to be applied), type CRG into the box, and press *Enter*.

4. Notice that the triangles have been colored in a way that makes the labels on triangle RGB unnecessary! Once you agree with this observation, press *Ctrl+L* (which is the same as clicking **View/Labels/Hide**), which makes all the labels disappear. (If you keep pressing *Ctrl+L*, the labels will return, by the way.)

5. Open the **Measurement** dialog, and obtain the measurements (all of them area ratios) CRG/RGB, CGB/RGB, and CBR/RGB. Close the dialog box.

6. Put the mouse into **Btns/Drag points** mode and use the left button to drag point C around inside RGB. Notice what happens to the displayed percentages. What is their sum? What happens when C gets close to R? when C gets close to $\overline{GB}$?

This diagram is an abstract model of the color mixing that takes place on a conventional television screen (such as the one you are looking at now). The screen is actually composed of thousands of tiny red, green, and blue dots. The figure at right represents a single three-dot team. By varying the brightness of the three neighboring dots, the illusion of a mixed color is created. Moving C towards a vertex corresponds to brightening that component. Each position of C corresponds to a color mixture. For instance, putting C at the centroid corresponds to mixing white (which does not actually show in this demo, of course). What positions of C correspond to colors that could be called *purple*?



In this way, colors can be described *numerically*, by giving the red-green-blue fractional distribution of area. For instance, the number triple $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is white, $(\frac{1}{2}, \frac{1}{2}, 0)$ is yellow, $(\frac{2}{3}, \frac{1}{3}, 0)$ is a shade of orange, $(0, \frac{1}{2}, \frac{1}{2})$ is cyan, etc. Very few colors actually have names, of course, and most names are fuzzy.

Part II: Ceva's Concurrence Theorem

7. Click **Other/AutoExtend** to activate this menu item. (It should now have a check next to it.) This enables the next construction: Click **Point/Intersection/Line-line**. Type CR into the first box, GB into the second, and press *Enter*. Intersection point A should appear on $\overline{GB}$. (Notice that the program has re-used a discarded label, for it was the first one available.) Type CG into the first box, BR into the second, and press *Enter*. Intersection point D should appear on $\overline{BR}$. Type CB into the first box, RG into the second, and press *Enter*. Intersection point E should appear on $\overline{RG}$. Close the dialog box.

8. Open the **Measure** dialog, **hide** or **delete** the ratios we have been looking at, and obtain the new ratios CGR/CGB, CBG/CBR, CRB/CRG, RD/DB, GE/ER, and BA/AG. Could you have anticipated the coincidences?

9. What is the product of the ratios CGR/CGB, CBG/CBR, and CRB/CRG? It is not necessary to ask the computer to calculate it.

10. What is the product of the ratios RD/DB, GE/ER, and BA/AG? It is not necessary to ask the computer to calculate it.

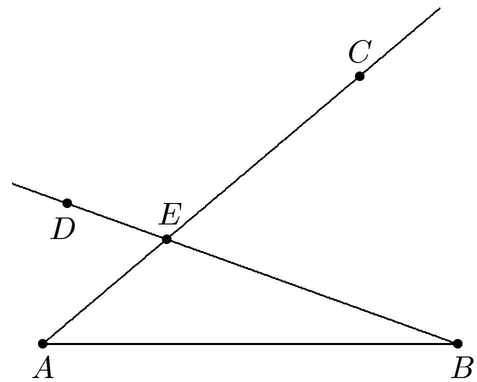
11. Notice that the lines $\overline{RA}$, $\overline{GD}$, and $\overline{BE}$ *are* concurrent at C (this is not a matter of speculation). Because C is a generic point inside a generic triangle, step 10 establishes Ceva's necessary condition for the concurrence of three lines drawn from the vertices of a triangle. State this as a theorem.

By the way, the converse of this statement is also true.

Constant Angle Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. (Items in boldface are meant to be clicked with the mouse.) This creates a small drawing window.

1. Click **Shape/Segment** and press *Enter*.



2. Click **Line/Angles/New**. Use this dialog box to draw two angles of specified size as follows: Check that AB is in the *initial ray* box, then type 60# into the *angle size* box (do not forget the #) and press *Enter*. Because the current value of # is probably 2/3, you should see an angle of about 40 degrees appear. Now type BA into the *initial ray* box, type 60# − 60 into the *angle size* box, and press *Enter*. You should see an angle of about 20 degrees open *clockwise* from $\overline{BA}$. (The first angle opened counterclockwise from $\overline{AB}$.) The figure now probably looks like the illustration. Before proceeding, you should think about the construction of these rays. It will help to know that the program treats positive angles in a counterclockwise sense, and negative angles in a clockwise sense. The figure was designed deliberately to have one of each.

3. Label the point where the two rays meet. The label chosen by the program will probably be E. (You can always change the program's labelling choices, by the way — just put the mouse into **Btns/Text** mode, right click any label you want to change, type your choice into the edit box, and press *Enter*. The only thing the program will not let you do is give the same label to two different points.)

4. What is the size of angle AEB? Make a prediction before you click open the **Measure** dialog to check your answer.

5. Click **Animate/Parameter values**. Click the arrow buttons to adjust the value of # upward or downward. For this example, the default range of values (from 0 to 1) is just what we want. Notice the path taken by the moving intersection E. It is easier to watch the complete range of motion if you click **Autoreverse**, but save your work before you do. Once the animation starts, the dialog box disappears, and the title bar of the drawing window reminds you to press a key when you want to stop. It is the only way.

6. To see the path of E, click **Animate/Tracing**. Type E into the *pen on* box, check that the *control* button for # is selected, and press *Enter*. The tracing takes place, then our construction is superimposed on top of it. Do you recognize this curve?

Fermat Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. (Items in boldface are meant to be clicked with the mouse.) This will create a small drawing window, to be used in Part I.

Part I

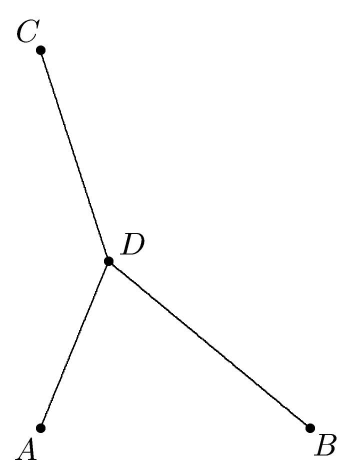
1. Click **Point/Grid** to activate the coordinate input box, which should show 0.0 in both the x and y boxes. Click **Label** to set $A = (0, 0)$. Type 5 into the x box and click **Label** again (or press *Enter*) to set $B = (5, 0)$. Finally, type 0 into the x box and 7 into the y box and click **Label** to set $C = (0, 7)$. Close the dialog box.

2. With the mouse in **Btns/Segment** mode, right-click a random (and therefore movable) point D onto the diagram. Open the **Measurement** dialog, type $DA + DB + DC$ into the edit box, and press *Enter*. Close the dialog.

3. Put the mouse into **Btns/Drag points** mode and then use the left button to drag D around the screen, watching the displayed measurement as you do. The object is to make the sum $DA + DB + DC$ as small as possible.

4. After some experimentation, it will eventually seem impossible to make the sum any smaller. This is because of the granularity of the screen. In a situation where increased accuracy is desired, it is possible to improve the resolution of the screen and keep searching. To do this, click **Btns/Coordinates**. Then point at D and click the *right* button. This enlarges the region around D by a factor of 2, and puts D at the center. (The magnification factor can be changed, by clicking **Other/Zoom factor**, but it is easier to just right-click a few times to obtain higher power.) If the displayed measurement ever gets lost, click **Btns/Home measurements** to put the item back in the upper left corner of the window. Now put the mouse into **Btns/Drag points** mode again and keep searching.

5. Depending on the precision you want to achieve, you may have to increase the number of displayed decimal places. To do this, click **Edit/Decimals**. Type the desired number of places into the edit box and press *Enter*. The display will change as soon as you start searching again.

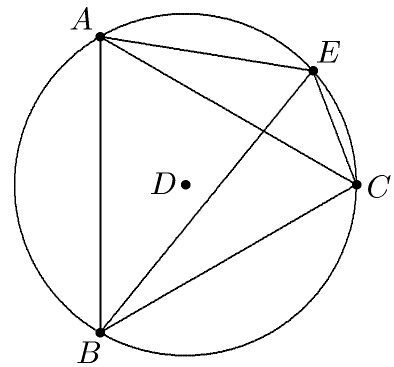


6. This zoom-and-drag process can be repeated until you finally get tired of doing it. Then it is time to write down the value of the sum, and also the coordinates of the point D that produces it. To see the coordinates, put the mouse back into **Btns/Coordinates** mode, point at D, and hold down the left button.

Part II

7. *Click the* **2-Dim** item on the top menu bar to open a second drawing window. If you are working in full-screen mode, you will have to reduce the size of the first window before you can open the second.

8. Click **Shape/Polygon/Regular**, type 3 into the *sides* box, and press *Enter* twice to see equilateral triangle ABC. Then click **Circle/Circumcircle**, check that ABC appears in the edit box, and press *Enter* to see the circumscribed circle of the triangle, whose center is labelled D.



9. With the mouse in **Btns/Segment** mode, right-click a random (movable) point E onto the circle between A and C, and then use the left button to make the connections $\overline{EA}$, $\overline{EB}$, and $\overline{EC}$.

10. Open the **Measurement** dialog, and ask for the sizes of $\angle AEB$, $\angle CEB$, and $\angle AEC$, one at a time. Also ask for EB and $EA + EC$. Close the dialog.

11. Put the mouse into **Btns/Drag points** mode and slide point E along the circle. Notice what happens to the displayed measurements. Could you have predicted what you see?

Part III

12. Open a third drawing window by clicking **2-Dim** on the main menu bar. It is often convenient to arrange multiple windows so that at least a small part of each window (the lower right corner is ideal) is always visible, even when the rest of the window is covered.

13. Repeat the construction of triangle ABC described in step 1 above (or save the figure in the first window, recall the figure in the third, and delete point D).

14. Join $\overline{CA}$ and $\overline{AB}$. Click **Shape/Polygon/Attach**, type 3 into the *sides* box, the list BA, AC into the *attach to* box, and press *Enter*. You should see equilateral triangles BAD and ACE appear.

15. With the mouse in **Btns/Segment** join mode, use the left button to draw $\overline{BE}$ and $\overline{CD}$. Then point at the intersection of these two segments and click the right button, labelling the intersection as F.

16. Put the mouse into **Btns/Coordinates** mode, point at F, and hold down the left button to see its coordinates. Compare these coordinates with the coordinates you found in step 6 above.

17. Use the **Measurement** dialog to obtain the value of $FA + FB + FC$, and compare this value with the minimal value you found in part I.

Fermat Lab

Part IIII

18. Open a fourth drawing window by clicking **2-Dim** on the main menu bar, and repeat steps 12-14 from part III.

19. Click **Circle/Circumcircle**, type the list ACE, BAD into the edit box, and press *Enter*. Two circles should appear, centered at F and G. Both circles go through A.

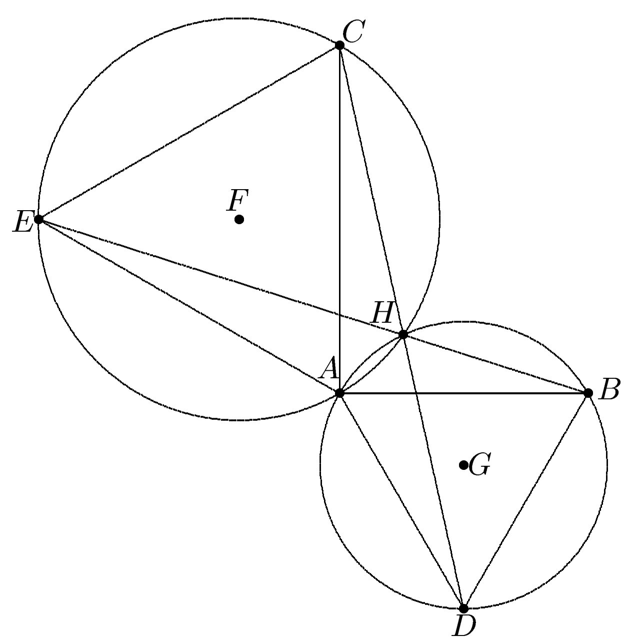
20. With the mouse in **Btns/Segment** mode, use the right button to label the unnamed intersection point of the two circles. Because the program always chooses the first available label, it should be H.

21. Put the mouse into **Btns/Coordinates** mode, point at H, and hold down the left button to see its coordinates. Compare these coordinates with the coordinates you found in step 16 above.

22. Use the **Measurement** dialog to obtain the lengths EB and CD, and compare these values with the value of $FA + FB + FC$ found in step 17 above.

23. Use the **Measurement** dialog to confirm that E, H, and B are collinear. Do the same for C, H, and D.

24. Can you explain these coincidences?



SSSS Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. (Items in boldface are meant to be clicked with the mouse.) This creates a small drawing window.

Part I: Construction of two quadrilaterals with sides 3, 4, 5, and 6

1. Click **Shape/Segment**, type 3 into the *length* box, and press *Enter*.

2. Click **Line/Angle/New**, check that AB appears in the *initial ray* box, type 109 into the *size* box, and press *Enter*. You should now see angle BAC, with $\overline{AC}$ making a 109-degree angle with $\overline{AB}$.

3. We want to mark a point on $\overline{AC}$ that is exactly 4 units from A. Here is one way to do it: Click **Point/Segment division**, and type AC into the *segment* box. Choosing the correct coordinate takes some thought. Remember that C has coordinate 1, but its distance from A is known only as AC. That is good enough, for the coordinate we want is therefore $4/AC$ (no matter what AC actually is). Therefore, type $4/AC$ into the *Mark at* box and press *Enter*. Point D will appear. (If you want, you can now use the **Measure** dialog to check that AD really is 4.)

4. Click **Circle/Radius-center** to draw two circles. First type D into the *center* box and 5 into the *radius* box, then press *Enter*. Circle DE should appear. Now type B into the *center* box and 6 into the *radius* box, then press *Enter*. Circle BF should appear.

5. The circles intersect in two places, which need labels. Put the mouse in **Btns/Segment** mode, and right-click the upper intersection. This point will probably get the label G and the other one will get H.

6. Thus there are at least *two* quadrilaterals that have 3, 4, 5, and 6 as lengths of sides. One is $ABGD$ and the other is $ABHD$. Rather than spend time looking at either one, however, we next refine the construction by building a *flexible hinge* at vertex A.

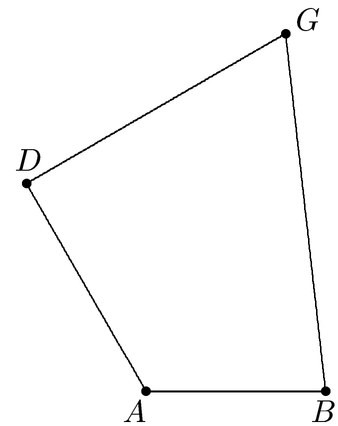
Part II: Construction of infinitely many quadrilaterals with sides 3, 4, 5, and 6

7. Start by **Edit/Deleting all**. Then simply repeat the preceding six steps, *except* that in step 2 the size of angle BAC should be entered as 180# (do not forget the # sign!).

8. Before exploring further, let us choose $ABGD$ as the preferred quadrilateral and clean up the traces of our construction: With the mouse in **Btns/Segment** mode, use the left button to make the connections $\overline{DG}$ and $\overline{BG}$. Now click **Edit/Circle delete**, type DE, BF into the edit box, and press *Enter*. The dialog box disappears, and so do the circles. Then click **Edit/Point delete**, type $CHEF$ into the box, and press *Enter*. All points except $ABDG$ are gone. Finally, click **Line/Extension**, type AD into the box, and press *Enter*. This removes an unwanted ray.

9. The symbol # stands for a *variable* numerical quantity. The default value (when the program starts) is $\# = 2/3$, so your figure probably looks something like the illustration at right, with a 120° angle at A.

10. To alter the value of #, click **Anim/Parameter values**. The resulting dialog box displays the current value of #, and it gives you a variety of ways of changing that value. You can simply type a new value into the edit box and press *Enter*, or you can use the mouse (left button) to slide the bar back and forth, or you can click either arrow at the end of the bar, which moves the bar one notch (out of a hundred) in the requested direction. You can click **Autoreverse** to let the bar slide back and forth by itself. *Once this animation starts, you will have to press a key to stop it.* This reminder is displayed on the title bar of the window.



11. The controls allow you to move the value of # back and forth between two extremes, initially 0 and 1. It is possible to change those extremes. *To enlarge the range*: Type a value into the box (say 1.3) that is outside the current range, and press *Enter*. When you slide the bar now, the #-value varies between 0 and 1.3. Stop at some intermediate value. *To narrow the range*: Click the left **reset** button. This makes the current #-value the left extreme. Slide the bar a bit to the right, and click the right **reset** button. This makes the current #-value the right extreme. If you now try to slide the bar, you will notice that the figure is restricted in its range of motion. Before continuing to the next item, reset the extremes to $0.4 \leq \# \leq 0.8$.

12. Open the **Measurement** dialog, type $ABGD$ into the box, and press *Enter*, to obtain the area of the quadrilateral. Also find (one at a time) the sizes of angles BAD and BGD. Press *Excape* to close the dialog box.

13. Return to varying the size of #. Remember that the variable # is controlling the size of angle BAD. As the angle changes size, the quadrilateral changes shape, and so the displayed measurements must change, too. Try to make the area of the quadrilateral as large as you can. By gradually narrowing the range of the slider, you can achieve a lot of precision. You may even want to click **Edit/Decimals** to increase the number of displayed decimal places.

14. When you have adjusted # to make the displayed area of $ABGD$ as large as you can, record the three displayed values. Then save the figure by clicking **File/Save** and choosing a file name.

Part III: Finding the largest convex quadrilateral with sides 3, 4, 6, and 5

15. Click **Edit/Delete all** to make a fresh start. Repeat the steps for part II, but interchange the positions of the sides of length 5 and 6. As in part II, vary the value of # until you have made your quadrilateral enclose the largest possible area. Record the three displayed values.

16. How does the maximal quadrilateral from part II compare with the maximal quadrilateral from part III? Are they congruent?

17. One (optional) way of comparing these two examples is to open a second **2-Dim** drawing window (click the main menu bar, which is only visible when your drawing windows are not in full-screen mode), then retrieve the first example by clicking **File/Old**. The two examples can then be viewed side-by-side.

Part IV: Get a new quadrilateral (a list of four consecutive edge lengths) from your teacher.

18. Repeat the construction used in the preceding examples, the goal being to select — from the infinitely many possibilities — the quadrilateral that encloses the largest area. Record your findings (area, together with the sizes of the two opposite angles BAD and BGD) on the board so that we can pool our results and look for a larger pattern.

Parabola Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. To get started, click **Shape/Segment**, then click **OK**. Segment AB should appear.

2. Click **Point/Grid**. Type the coordinates $x = 0.5$ and $y = 0.3$ into the boxes. (By the way, you can use the *Tab* key to move from one box to the other, or else click with the left button. Tabbing always highlights existing text, which means that it will be automatically replaced by whatever you type.) Click **Label** to mark point C at these coordinates. You can press *Escape* to close the dialog box, which removes the coordinate axes.

3. At any time, you can undo your last step by clicking **Edit/Undo**.

4. Click **Point/Segment division**. We want to mark a movable point on segment AB. To do this, type # (which stands for a variable quantity) into the edit box, check that the *segment* box shows AB, then click **Mark at**. Point D should appear on $\overline{AB}$.

5. We want to connect C to D, so click **Line/Segments**. Type CD (lower-case letters are fine, by the way) and press *Enter* (or click **OK**). The segment should appear.

6. We want to draw the perpendicular bisector of the segment we just drew. Click **Line/Perpendicular/Bisector** . Type CD into the edit box and press *Enter* (or click **OK**). Line EF should appear, with E the midpoint of $\overline{CD}$.

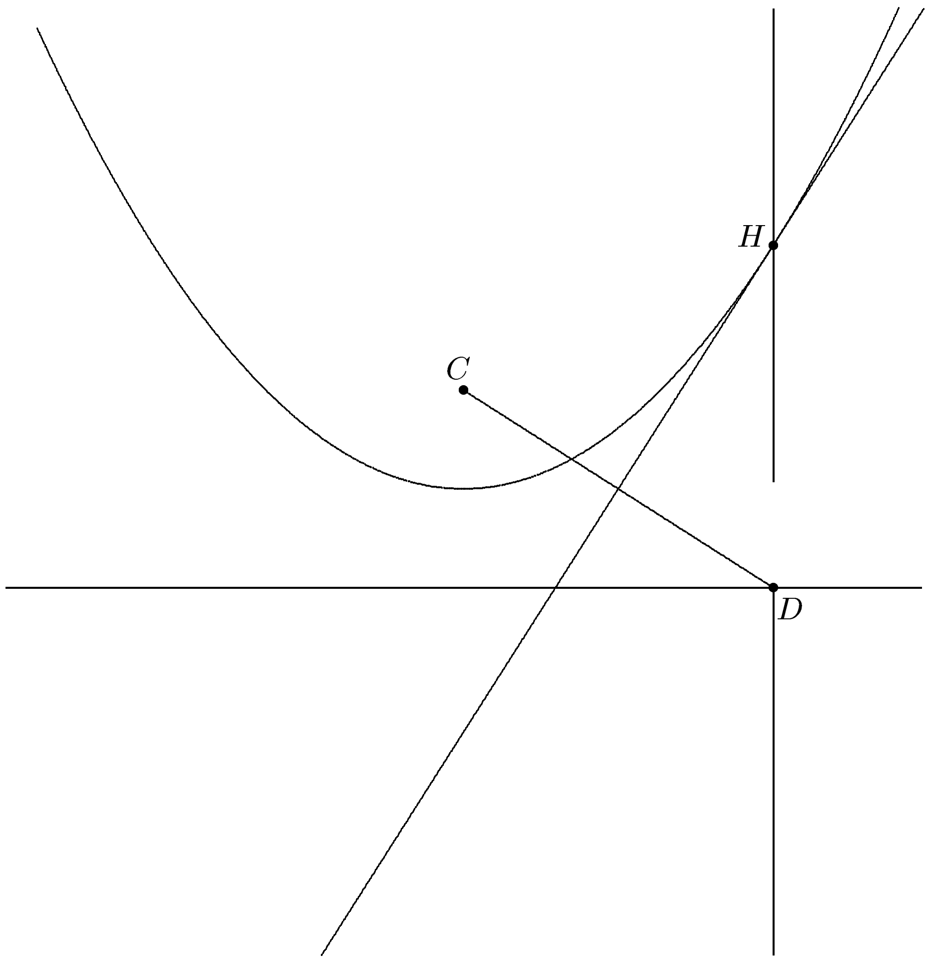
7. We wish to draw the line that is perpendicular to $\overline{AB}$ and goes through D, so click **Line/Perpendiculars/General**. Type AB into the *perpendicular to* box and D into the *through* box and press *Enter* (or click **Mark**). Line DG should appear. The view frame will probably be repositioned because of this new point. Close this dialog box.

8. We want to label the point where the two perpendiculars intersect. There are two ways this can be done. If the mouse is in **Btns/Segment** mode, just click the intersection point with the *right* button. The label H should appear. (The other approach: Click **Point/Intersection/Line-line**. Type the segment names DG and EF into the boxes and click **Mark**. Label H should appear. You can press *Escape* to close the dialog box.)

9. Click **Animate/Parameter values**. This creates a scroll-bar control that allows you to vary the position of point D on the line AB. The various positions are described by the single coordinate #, which has been set up so that 0 corresponds to A and 1 corresponds to B. You can change the position of D by typing a new value for # and pressing *Enter*, or by using the left mouse button to click and drag the scrolling button, or by clicking the left and right arrows. Try it. When the figure responds, the position of point H changes. Keep your eye on the path it follows. Do not click the two **Reset** buttons, unless you know what they do.

10. You can also put the scroll-bar in autopilot mode by clicking the **Autoreverse** button. Notice that the dialog box disappears, and that a new caption appears on the drawing window: To stop the animation, you will have to press Q.

11. To see the parabolic path of H, click **Animate/Tracing**. Put the *pen on* vertex H, check that parameter # is the *control*, then click **OK**. The parabolic curve should appear, with our construction superimposed:



Questions: With the exception of point H, all the points on the perpendicular bisector of $\overline{CD}$ are closer to $\overline{AB}$ (the *directrix*) than to C (the *focal* point). Explain why. This shows that the perpendicular bisector of $\overline{CD}$ is tangent to the parabola (at H). Explain why.

Orthocenter Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. (Items in boldface are meant to be clicked with the mouse.) This creates a small drawing window.

1. Request a random triangle, hereafter referred to as *Triangle I*.

2. Construct *Triangle II*, which is the unique triangle whose midpoints are the vertices of Triangle I.

3. Construct the perpendicular bisectors of the sides of Triangle II. They are *concurrent* at a point that is called the ____________________ of Triangle II.

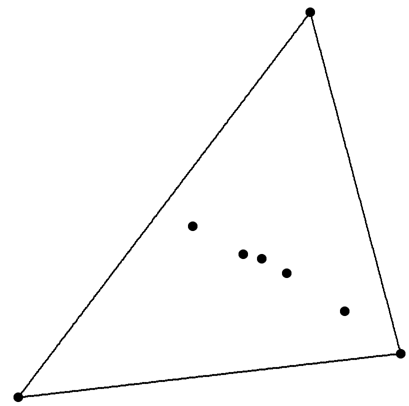
4. The perpendicular bisectors drawn in step 3 are also special lines for Triangle I. They are the ______________________ of Triangle I. Their point of concurrency is called the *orthocenter* of Triangle I.

5. Use the mouse to drag the vertices of Triangle I around the screen, watching what happens to the orthocenter. In particular, is it clear which Triangles I enclose their orthocenters? When the movement of Triangle I causes the orthocenter to drift outside the triangle, where does it leave the triangle?

The Five Concurrencies Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. Request a random triangle. The next five steps ask you to construct the five points of concurrency we have talked about. You can do them in any order. At the end of the project, only the five points and the triangle should be left on the screen, so you may want to clean up (that means delete things you no longer need) as you go. It is also recommended that you *save* your work as you go, just as you would if you were using a word processor. The finished product will look something like the figure at right.



2. Construct the *centroid* of your triangle, which is the concurrence of the medians.

3. Construct the *circumcenter* of your triangle, which is the concurrence of the perpendicular bisectors of the sides.

4. Construct the *Fermat point* of your triangle, which is the point that makes the sum of the distances to the three vertices as small as it can be. As we have seen, it is also a point of concurrence.

5. Construct the *incenter* of your triangle, which is the concurrence of the angle bisectors.

6. Construct the *orthocenter* of your triangle, which is the concurrence of the altitudes.

7. Use the mouse to move the vertices of your triangle around the screen, watching what happens to the five special points. It is a challenge to recognize which is which. Which of the five points are always enclosed by the triangle? You may notice that three of the five points always lie on a single line. Can you identify which three? The line is called the *Euler line* of the triangle.

Perspective Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse. Remember to save your figure as you proceed — just in case.

Part I: Two-point perspective drawing of a box

1. Click **Shape/Segment** and press *Enter*. Segment AB appears.

2. Click **Line/Perpendicular/Bisector**, type BA into the box (notice the order of letters), and press *Enter*. The perpendicular bisector $\overline{CD}$ of $\overline{AB}$ appears, with C the midpoint of $\overline{AB}$ and D somewhere below.

3. With the mouse in **Btns/Segment** mode, right-click two points E and F onto $\overline{AB}$, with E to the left and F to the right. Right-click two points G and H onto $\overline{CD}$, with G above H. Use the left button to connect E and F each to G and H (four segments needed). Click **Other/Autoextend**, so that segments will extend when necessary.

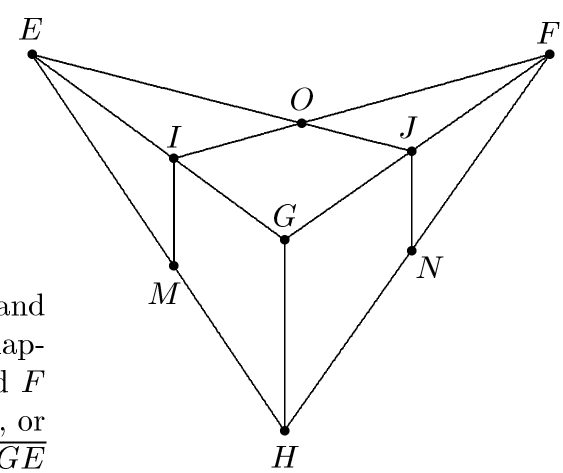
4. Right-click point I onto $\overline{GE}$ and point J onto $\overline{GF}$.

5. Click **Line/Parallels**, type GH into the *parallel to* box, I into the *through pt* box, and press *Enter*. Then type J into the *through pt* box and press *Enter*. Parallels $\overline{IK}$ and $\overline{JL}$ should appear. Right-click the label M onto the intersection of $\overline{IK}$ with $\overline{HE}$, then right-click the label N onto the intersection of $\overline{JL}$ with $\overline{HF}$.

6. Use the left button to make the connections $\overline{IF}$ and $\overline{JE}$.

7. Click **Point/Intersection/Line-line**, type IF into the first box, JE into the second box, and press *Enter*. Label O should appear at the intersection of $\overline{IF}$ and $\overline{JE}$. Close the dialog.

8. The box is now complete, so let's clean up the figure. Click **Edit/Delete pt** to remove points $ABCDKL$. Click **Edit/Delete segment** to remove $\overline{EF}$. Click **Line/Extensions**, type the list IM, MI, JN, NJ, GH, HG, and press *Enter*. The figure should now look somewhat like the illustration.



9. Put the mouse in **Btns/Drag points** mode, and experiment with the drawing, noticing what happens when you slide the vanishing points E and F horizontally, the box corners G and H vertically, or the box corners I and J back and forth along $\overline{GE}$ and $\overline{GF}$. These six points are the only ones that are movable.

10. Use the **Measure** dialog to confirm that quadrilateral $GJOI$ is not special in any way (regardless of what in reality the quadrilateral represents).

11. Find the center of face $GJOI$ by intersecting its diagonals. Join this central point to the vanishing point at F, then label the intersection of this segment with $\overline{JO}$. This point may *represent* the midpoint of a real segment, but it is *not* the midpoint of $\overline{JO}$ that appears on the screen. Make a simple measurement to confirm this.

Part II: One-point perspective drawing of railroad tracks

12. Click **Edit/Delete all** to make a fresh start. Click **Shape/Segment** and press *Enter* to make horizontal $\overline{AB}$ appear.

13. Put the mouse into **Btns/Segment** mode and right-click a random point C into the upper right corner of the drawing window. (This is the vanishing point.) Use the left button to make the connections $\overline{AC}$ and $\overline{BC}$. Right-click a random point D onto $\overline{AC}$.

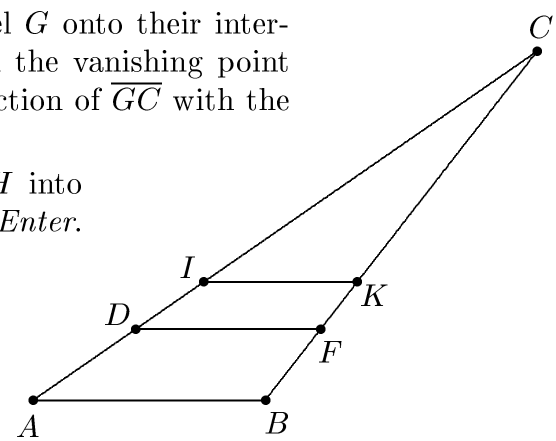
14. Click **Line/Parallels**, check that AB shows in the *parallel to* box, type D into the *through pt* box, and press *Enter*. Parallel $\overline{DE}$ should appear. Close this dialog.

15. Right-click the label F onto the intersection of $\overline{DE}$ and $\overline{BC}$. This places $\overline{AB}$ and $\overline{DF}$ as the first two crosspieces of our railroad tracks.

16. Now put in the next crosspiece: Use the left button to draw the diagonals $\overline{AF}$ and $\overline{BD}$, then right-click the label G onto their intersection. Use the left button to connect G with the vanishing point C, then use the right button to label the intersection of $\overline{GC}$ with the crosspiece $\overline{DF}$. Draw $\overline{HB}$.

Click **Point/Intersection/Line-line**, type BH into the first box, AC into the second box, and press *Enter*. Label I should appear. Close this dialog.

Use the **Line/Parallels** dialog to draw the line that is parallel to $\overline{AB}$ and that goes through I. Then use the right button to label K, the intersection of this line with $\overline{BC}$.



17. Three crosspieces are now in place. Use the same method to build fourth and fifth crosspieces into the diagram. Then put the mouse into **Drag points** mode and play with the perspective.

18. Examine the sequence of numbers that begins BF, FK, and that continues using the two new lengths you have just marked on $\overline{BC}$. Is there any pattern to this list? How about the list that begins BC, FC, KC, ...?

19. How would you make a *two-point* perspective drawing of railroad tracks?

Minimization Lab I

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse. Remember to save your figure as you proceed — just in case.

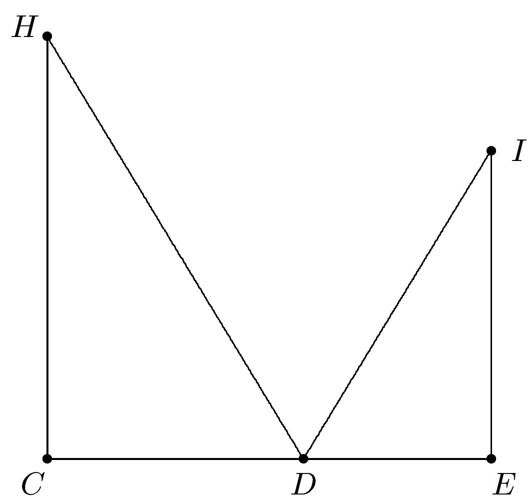
1. Click **Shape/Segment** and press *Enter*. Segment AB appears.

2. With the mouse in **Btns/Segment** mode, right-click three movable points C, D, and E onto $\overline{AB}$, with C to the left and E to the right.

3. Click **Lines/Perpendiculars/General**, check that AB is in the *perp to* box, type C into the *through pt* box, and press *Enter*. Then type E into the *through pt* box and press *Enter* again. You should see $\overline{CF}$ and $\overline{EG}$.

4. Right-click point H onto $\overline{CF}$ and point I onto $\overline{EG}$. These points represent the tops of two poles. Use the left button to draw the connections $\overline{HD}$ and $\overline{DI}$. These represent cables running to a point on the ground.

5. Time to clean up the figure. **Edit/Delete points** $ABFG$. Click **Line/Extensions**, type the list CH, HC, EI, IE into the box, and press *Enter*. The figure should now look something like the illustration.



6. Open the **Measurement** dialog and obtain the combined length $HD + DI$ of the cable. Close the dialog.

7. Put the mouse into **Btns/Drag points** mode and slide the anchor point D back and forth along the ground. The object is to make the displayed length $HD + DI$ as small as possible.

8. When you have found the optimal position for D, open the **Measurement** dialog again and ask for the sizes of angles CDH and EDI. Hmmm.

9. If you change the height of either pole (by using the left button to drag H or I vertically), then D will not automatically adjust its position. If you change the space between the poles (by using the left button to drag C or E horizontally), then D will not automatically adjust its position. In either case, D no longer provides the smallest possible value for $HD + DI$. (Try this and see.) Therefore our search gives information about only *one example at a time*. Invent an alternate procedure — one that constructs the optimal D in a way that applies no matter how we alter the heights of the poles or the space between them. In other words, when C, E, H, or I are moved, the anchor point D will automatically adjust its position.

Minimization Lab II

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse. Remember to save your figure as you proceed — just in case.

1. With the mouse in **Btns/Segment** mode, right-click three random points A, B, and C onto the screen. Then click **Edit/Coordinates** to place them more precisely. **Move** A to $(0, 10)$, **move** B to $(20, 0)$, and **move** C to $(0, 0)$. Close the dialog box. Press *Ctrl+W* to recenter the figure. Use the left button to make the connection $\overline{BC}$. Right-click a random movable point D onto $\overline{BC}$. Connect $\overline{DA}$.

2. Open the **Measurement** dialog and obtain the value of $AD/30 + DB/50$. This value represents the total *time* it takes to travel from A to D at 30 uph and from D to B at 50 uph. Close the dialog.

3. With the mouse in **Btns/Drag points** mode, slide D along $\overline{BC}$, trying to make the displayed measurement as small as possible.

4. Once you have made $AD/30 + DB/50$ as small as you can, recover the coordinates of the optimal position for D: Put the mouse into **Btns/Coordinates** mode, point at D, and hold down the left button.

5. Let's change the problem now, by moving the station B further down the road. Following the procedure of step 1 above, **move** B to (30,0). Point B should jump to the right, and the displayed measurement will change. Press *Ctrl+W*. Now put the mouse back into **Btns/Drag points** mode and see where the optimal position for D is. Hmmm.

Minimization Lab III – Snell's Law

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse. Remember to save your figure as you proceed — just in case.

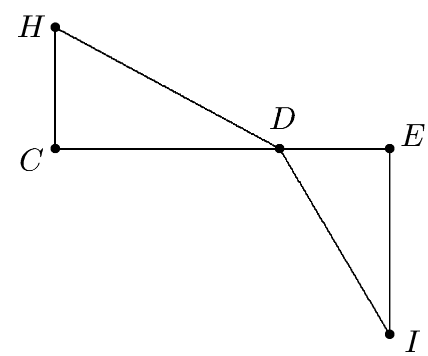
1. Click **Shape/Segment** and press *Enter*. Segment AB should appear. To allow segments to be extended whenever necessary, click **Other/Autoextend**.

2. With the mouse in **Btns/Segment** mode, right-click three movable points C, D, and E onto $\overline{AB}$, with C to the left and E to the right.

3. Click **Lines/Perpendiculars/General**, check that AB is in the *perp to* box, type C into the *through pt* box, and press *Enter*. Then type E into the *through pt* box (leave AB as is) and press *Enter* again. You should see $\overline{CF}$ and $\overline{EG}$.

4. Right-click point H onto $\overline{CF}$, and point I onto the part of $\overline{EG}$ that is below $\overline{AB}$. Use the left button to make the connections $\overline{HD}$ and $\overline{DI}$.

5. Time to clean up the figure. **Edit/Delete points** $ABFG$. Click **Line/Extensions**, type the list CH, HC, EI, IE into the box, and press *Enter*. The figure should now look something like the illustration.



6. Open the **Measurement** dialog box and put the sum $HD/10 + DI/20$ on display. Invent a story to go with this measurement. Close the dialog.

7. Drag D back and forth along $\overline{CE}$, trying to make the displayed measurement as small as you can.

8. Once D is in its optimal position, open the **Measurement** dialog and ask for the values of the ratios DE/DI and DC/DH. (Or, if you are familiar with trigonometry, you can ask for $[sin](\angle EID)$ and $[sin](\angle CHD)$. Notice that the program expects special punctuation when functions are used.) You should see a simple relationship between these values. Hmmm.

9. When you think that you have identified the pattern, test your guess (it is *not* expected that you find a proof) by inventing a new problem — remember that you can move C, H, E, and I, and the numbers 10 and 20 are also arbitrary.

Two-Mirror Lab

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

Part I

1. To get started, click **Shape/Segment**, type 20 into the *Length* box, and click **OK** (or press *Enter*). Horizontal $\overline{AB}$ should appear on the screen.

2. Click **Shape/Triangle/SSS**, type the lengths 5, 4, and 3 into the three edit boxes, and press *Enter*. Triangle CDE should appear somewhere on the screen.

3. Put the mouse in dragging mode, by clicking **Btns/Drag point**. Slide CDE to a position below the segment.

4. To create the mirror image of the triangle, click **Transform/Mirror**. Type AB into the *mirror* box and CDE into the *apply to* box. Click **OK** (or press *Enter*) and the image triangle $C'D'E'$ should appear on the other side of the mirror.

5. The mirror diagram needs an observer. Put the mouse into drawing mode by clicking **Btns/Segment**, then point at a place somewhere to the left of the triangle and click the *right* button. Point F should appear.

6. Use the left mouse button to draw out the connection $\overline{FC'}$. This corresponds to the observer F looking at the corner C'.

7. Point at the intersection of the sightline $\overline{FC'}$ and the mirror $\overline{AB}$. Click the right mouse button. The intersection should now be labelled G. Use the left button to draw out the connection $\overline{GC}$. What does this segment represent? (If the mirror does not intersect the sightline, then first use the mouse to reposition the observer or the triangle.) See the diagram two pages ahead. What does the broken path from C to G to F represent? How does its length compare to that of $\overline{C'F}$?

8. Click open the **Measurement** dialog, and request the sizes (one at a time) of the angles AGF and BGC. Close the dialog by pressing *Escape*. The measurements should be showing in the drawing. Explain the coincidence of values.

9. Put the mouse in point-dragging mode, by clicking **Btns/Drag point**. Drag the observer F and the triangle CDE to new locations, and notice what happens to the rest of the diagram and to the measurements.

Two-Mirror Lab

Part II

10. Remove the work done in steps 6 through 9. One way to do this is to start with a clean slate and redo steps 1 through 5. Another way is to click **Edit/Undo** three times, then open the **Measurement** dialog, delete both items, and press *Escape*. The screen should show just the mirror $\overline{AB}$, the triangle CDE, its image $C'D'E'$, and the observer F.

11. It is time to add a second mirror to the diagram, perpendicular to the first. One way to do this is to click **Transf/Rotate**, type -90 into the *angle* box, type A into the *center* box, leave the *dilation* box alone, and type only B into the *apply to* box. Then press *Enter*. A new point B' should appear. To complete the construction of the mirror, click **Line/Segment**, type AB' into the box, and press *Enter*. Mirror $\overline{AB'}$ should now be visible. Do you see why it was necessary to type -90 rather than 90 into the *angle* box?

12. Another mirror means more images, so click **Trans/Mirror**, type AB' into the *Mirror* box, type $CDEC'D'E'$ into the *apply to* box, and press *Enter*. You should see two new triangular images, $C''D''E''$ and $C^{\sim}D^{\sim}E^{\sim}$. Do you see what makes the image $C^{\sim}D^{\sim}E^{\sim}$ *different* from $C'D'E'$ and $C''D''E''$?

13. There are now *three* images of C on the screen. Use the left mouse button to connect F to the image point $C^{\sim}$ that appears in the upper left corner. This connection will intersect *one* of the two mirrors, so right-click this intersection point to label it G. Use the left button to connect G to the image of C that belongs to the *other* mirror, then right-click the resulting intersection point to label it H. There is now an intersection marked on each mirror. Use the left button to connect C to H. See the diagram on the next page. There is now a broken three-segment path that leads from C to H to G to F— what does this path represent?

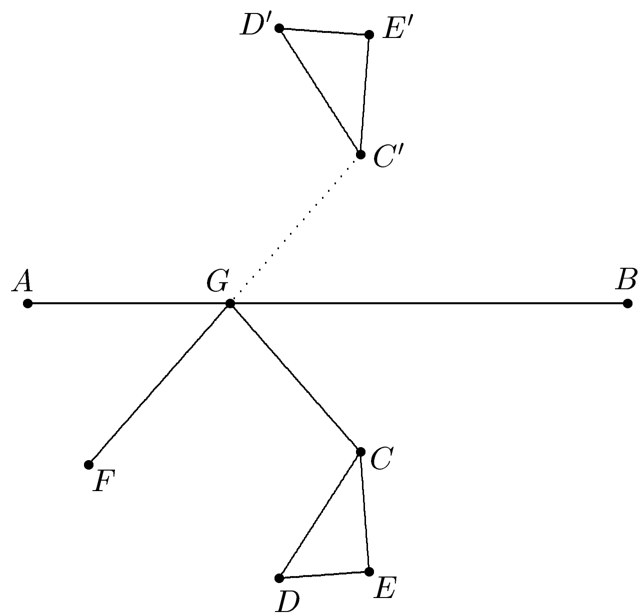
14. Put the mouse into **Btns/Drag points** mode, drag the observer around, and watch what happens. In particular, find a position for F that makes $\overline{FC^{\sim}}$ intersect the other mirror. You may be dismayed to see points G, H, and the recently constructed segments disappear!

15. Once G and H have disappeared, put the mouse back into **Btns/Segment** mode, and right-click the new intersection point to label it I. Use the left button to connect I to the image of C that belongs to the *other* mirror, then right-click the resulting intersection point to label it J. Use the left button to connect C to J, completing a broken, three-segment path that leads from C to J to I to F.

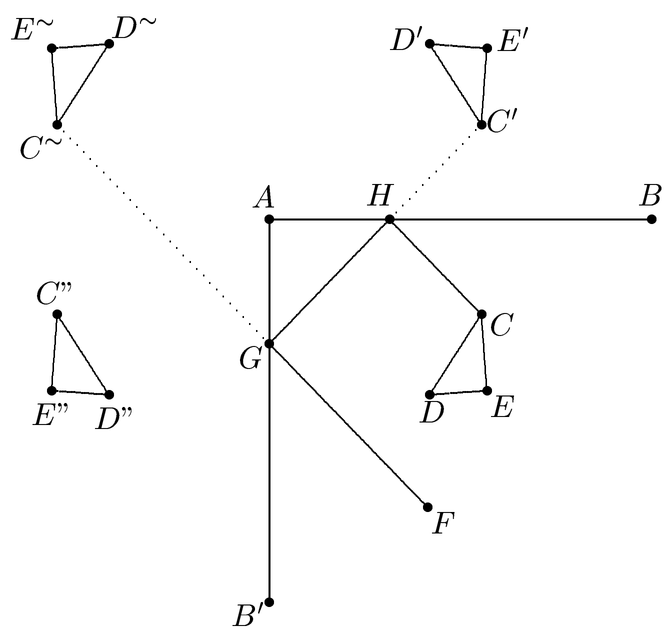
16. Finally, drag the observer inside the triangle CDE. This simulates you looking at your right ear, while seated in front of an arrangement of two perpendicular mirrors. The light that reaches your eye from your right ear must strike both mirrors, one before the other. If you were to focus your attention on your *left* ear D, the light you would see would also have struck both mirrors, but in the opposite order.

Two-Mirror Lab

Part I



Part II



Some Constructions

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

SSS Construction — lengths 4, 5, and 6

1. To get started, click **Shape/Segment**, type 4 into the *Length* box, and click **OK** (or press *Enter*). Horizontal $\overline{AB}$ should appear on the screen.

2. We need to draw two circles, so click **Circle/Radius-Center**. Type A into the *center* box, 5 into the *radius* box, and press *Enter*. Type B into the *center* box, 6 into the *radius* box, and press *Enter*. If the diagram is not all visible on the screen, click **View/Window** (or press *Ctrl+W*) to re-center everything. Press *Escape* to close the dialog box.

3. The circles intersect in two places. With the mouse in **Btns/Segment** mode, right-click either of the intersection points and both will receive a label. Use the left button to draw connections from one of the intersection points to A and to B. The 4-5-6 triangle is complete. When you are done looking at it, click **Edit/Delete all** to clear the screen for the next construction.

SAS Construction — length 4, angle 37 degrees, length 5

4. To get started, click **Shape/Segment**, type 4 into the *Length* box, and click **OK** (or press *Enter*). Horizontal $\overline{AB}$ should appear on the screen.

5. Click **Line/Angles/New**. The *initial ray* box should already show AB, so type 37 into the *angle size* box, and press *Enter*. Ray AC should appear. Press *Escape* to close the dialog box.

6. We need to find the point on ray AC that is 5 units from A. Here is one way to do it: Click **Point/Segment division**, type AC into the *segment* box, the coordinate $5/AC$ into the *Mark at* box, and press *Enter*. Point D will appear. Press *Escape* to close the dialog box. If you want to check that the length of segment AD really is 5, click **Measurement**. Use the left mouse button to draw the segment joining B to D. Triangle BAD is the desired triangle. When you are done looking at it, click **Edit/Delete all** to clear the screen for the next construction.

ASA Construction — angle 41 degrees, length 5, angle 82 degrees

7. To get started, click **Shape/Segment**, type 5 into the *Length* box, and click **OK** (or press *Enter*). Horizontal $\overline{AB}$ should appear on the screen.

8. We need to draw two rays, so click **Line/Angles/New**. The *initial ray* box should already show AB, so type 41 into the *angle size* box, and press *Enter*. Ray AC should appear. Now type BA into the *initial ray* box, -82 into the *angle size* box (notice the sign!), and press *Enter*. Ray BD should appear. Press *Escape* to close the dialog box. If the diagram is not all visible on the screen, click **View/Window** (or press *Ctrl+W*) to re-center everything.

9. Use the right mouse button to label the intersection of rays AC and BD. Triangle ABE is the desired triangle. When you are done looking at it, click **Edit/Delete all** to clear the screen for the next construction.

SSA Construction — lengths 8 and 7, angle 50 degrees

10. To get started, click **Shape/Segment**, type 8 into the *Length* box, and click **OK** (or press *Enter*). Horizontal $\overline{AB}$ should appear on the screen.

11. Click **Line/Angles/New**. The *initial ray* box should already show AB, so type 50 into the *angle size* box, and press *Enter*. Ray AC should appear. Press *Escape* to close the dialog box.

12. We need to draw a circle, so click **Circle/Radius-Center**. Type B into the *center* box, 7 into the *radius* box, and press *Enter*. A circle appears, with a random point D marked on it. Press *Escape* to close the dialog box.

13. Notice that the circle meets the ray AC in two places. Unlike the *SSS* case discussed above, these two points determine essentially different triangles. With the mouse in **Btns/Segment** mode, right-click either of the intersection points and the labels E and F will appear. Use the left button to make the connections $\overline{BE}$ and $\overline{BF}$. The two triangles ABE and ABF both fit the *SSA* description.

14. Use the **Measurement** dialog to find the sizes of the angles (one in each triangle) that are opposite $\overline{AB}$. Do you notice anything special about the answers?

15. If the *SSA* data had been different, it might have happened that *no* triangles fit the given description. Explain. Or, it might have happened that only *one* triangle fit the given description. Explain.

Using Macros

Every figure produced by *Wingeom* resides in the computer as a sequence of instructions, which have been applied to some initial configuration of vertices, lines, and circles. In principle, such a sequence of instructions ought to be applicable to another initial configuration (so long as it is compatible with the first), without the need of re-entering any of those instructions. To employ a construction in this way is to treat it as a *macro*. Here is a simple illustration.

Inscribing a square in a right triangle

1. Click **Point/Grid** and define the points A = (6,0), B = (0,0), and C = (0,8). The actual coordinates are not significant, but the intention is to make B a right angle. Press *Escape* to close the dialog box.

2. Use the left mouse button to draw the segments AB, BC, CA.

3. Now the macro begins: Click **Line/Angles/Bisect old**, type ABC into the edit box, and press *Enter*. The bisector BD should appear. Press *Escape* to close the dialog box. Use the right mouse button to make E the intersection of $\overline{AC}$ and ray BD.

4. Click **Line/Parallels**. Type E into the *through point* box and AB into the *parallel to* box, and press *Enter*. Line EF appears. Keeping E in the *through point* box, type BC into the *parallel to* box, and press *Enter* again. Line EG appears. Press *Escape* to close the dialog box.

5. Use the right mouse button to make H the intersection of line EF and segment BC, and I the intersection of line EG and segment AB.

6. Time to clean up: Click **Edit/Point delete**, type DFG into the edit box, and press *Enter*. Click **Edit/Segment delete**, type BE into the edit box, and press *Enter*. Click **Line/Extensions**, type the list HE,EH,IE,EI into the edit box, and press *Enter*. You should now see square $BIEH$ inscribed in the original triangle ABC.

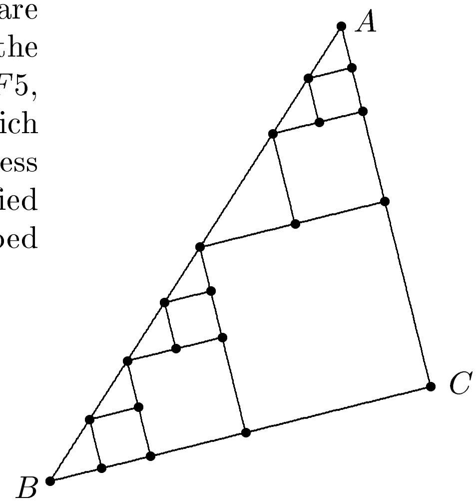
7. This completes the macro (including the cleanup step). When this figure is saved, it will be preserved for future use. It is convenient to specify our intentions now: Click **Edit/Macro define/Start and stop**. A list box appears, containing the complete history of the construction. Notice that the first few items in the list are devoted to creating the triangle ABC – they are *not* part of the macro. Find the angle-bisection step where our procedure truly begins (it probably appears as item 6 in the inventory). Highlight this item by clicking it, then click the **Start** button. To complete the macro definition, scroll through the list until you find the very last step, highlight it, and click the **Stop** button. The caption on the dialog box should display your selections. Click **OK** to confirm your choices and close this dialog box. Although it is not necessary in this example, click **Edit/Macro define/Variables**. The edit box probably already displays A,B,C, which are indeed exactly the points that were used to define the procedure. You can confirm this list by clicking **OK** to close the dialog box.

8. Now **File/Save** this figure. Call it *testmac*. Close the drawing window, ready for a fresh start. We are next going to use the macro a few times.

9. Click **2-Dim** to get a new drawing window, then click **Shape/Random/right triangle**. Triangle ABC will appear, and C will probably be the right angle.

10. Click **Other/Open macro**. A new drawing window appears, with a special macro caption. On its menu bar, click **File/Open**, type *testmac* into the edit box, and press *Enter*. The construction we just put away should reappear in the macro window.

11. Arrange and size the two windows so that both are visible. The item **Other/Apply macro** appears on the macro window's menu bar. Click it (or just press $F5$, which is easier). Type ACB into the edit box, which tells the computer to apply the inscribed-square process to the new right triangle, whose right angle is identified by C. Press *Enter*, and you should see the inscribed square appear.



12. Press $F5$ again, and apply the macro to another (smaller) right triangle in the main figure. To do this, just type the three-letter name of the triangle into the edit box, being sure that the second label you type marks the right angle.

13. Repeat this procedure a few more times. This will produce a figure that looks something like the illustration. To hide the many labels in the drawing, click **View/Labels/Hide** on the drawing window's menu bar (or just press *Ctrl+L*, which is easier).

14. What would happen if we tried to apply the macro to a triangle that did not have a right angle? What kind of figure would be inscribed instead of a square?

15. On the macro window's menu bar, click **Edit/Header**. Type *macro to inscribe rhombus in triangle ABC* and press *Enter*, then click **File/Save**. This explanatory remark has now been saved along with the macro. You can view it by clicking **Other/Lists/History**.

Lattice Points and Circles

0. Start *WinGeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, we use its menu bar. Items in boldface are meant to be clicked with the mouse.

1. With the mouse in **Btns/Segment** mode, right-click a random point A onto the screen.

2. Click **Circle/Center-Radius**, type # into the *radius* box, and press *Enter*. Circle AB should appear. The character # stands for a variable numerical quantity, which is probably 2/3 now. It will be changed soon. You can close the dialog box by pressing *Escape*.

3. Click **View/Grid**. This turns on the coordinate axes that are usually invisible. You can now estimate the size of # by using the scale on the axes. For this lab, we are mainly interested in *lattice points*, which are points whose coordinates are integers. Click **View/Dots** to turn them on, and press *Ctrl+E* a few times (which is the same as clicking **View/Expand**), until there are about twenty lattice points in sight. Click **View/Grid** to turn off the axes, which are not actually needed.

4. Put the mouse in **Btns/Drag points** mode. You can drag the circle around the screen by pointing at its center A and holding the left button down while the moving the mouse. (If you try to drag the circle by pointing at B, the whole figure moves.)

5. The purpose of this investigation is to discover the possible sizes for a circle that encloses *exactly three* lattice points. Points that lie *on* the circle are *not* enclosed, by the way. The current circle is probably only large enough to enclose two lattice points. Click **Animate/Parameters**. The resulting dialog box displays the current value of the radius #, and gives you various ways to change that value. The simplest way is to type a new number into the edit box, say 0.9, and then press *Enter*. The circle will respond immediately, changing its size (but not its center).

6. Continue to drag the circle around the screen, trying to position it so that there are exactly three lattice points inside. You should find that this is now possible. After some experimentation, you are ready to tackle the problem of finding the *largest radius* that allows a circle to enclose exactly three lattice points. This will consist of trial and error at first (dragging the circle and resizing it gradually), but you will eventually recognize this as a circumcircle problem, whose solution can be constructed exactly.

7. The same question can be asked for any number of lattice points: What is the radius of the largest circle that encloses *no* lattice points? that encloses *one* lattice point? that encloses *two* lattice points? Do the radii of these special circles increase as the number of enclosed lattice points increases?

8. Would it make sense to ask for the *smallest* circle that encloses a specified number of lattice points? Explain.