Wingeom Activities

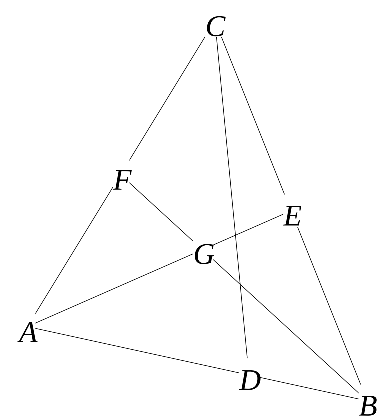
The following activities are designed to acquaint new users with the features of the program Wingeom. The first eight pages are not keyed to any particular mathematical problems, but there are opportunities for mathematical discussion. The four labs that follow were written to accompany certain problems that appear in our departmental text materials. These problems have been appended at the end as a supplement.

Introductory Activities

0. Start *Wingeom* by double-clicking its icon (found in the *Peanut* program group), or by clicking the icon once and pressing *Enter*. Click the **2-Dim** item on the main menu bar. This will create a small drawing window. From now on, you will be using its menu bar. Items in boldface are menu items or dialog buttons that are meant to be clicked with the mouse. Keyboard input, such as *Enter*, *Escape*, and *Ctrl+W*, is generally written in italics.

1. The left and right mouse buttons perform different functions, depending on the item that is checked on the **Btns** menu. The primary drawing mode is **Btns/Segment**. Check this item by clicking it, then point at three random places in the drawing window and click the right button. This labels three points *A*, *B*, and *C*. Point at *A*, hold down the left button, slide the pointer to *B*, and release the button. Segment *AB* now appears on the screen (unless the button release was not close enough to *B*). Repeat the process to draw segments *BC* and *CA*.

2. Right-click a point on segment *AB*. Its assigned label will be *D*, for the program always selects the first available label for a new point. Right-clicking is one way of marking a point on a segment. Here is another: Click **Point/Segment division**, type the list *BC,CA* into the *segment* box, notice that the *Mark at* box shows the coordinate 0.5, and press *Enter*. Labels *E* and *F* appear at the midpoints of segment *BC* and *CA*, respectively. You can close this dialog box by simply pressing *Escape* (or use the system menu). Use the left button to draw segments *CD*, *AE*, and *BF*, as in step 1. Point at the intersection of *AE* and *BF* and click the right button. The intersection point should now be labelled *G*.



3. Click **Btns/Drag points** to put the mouse into a different mode. Point at *B*, hold down the left button (notice that the cursor arrow disappears and the label color changes), and slide the mouse. Point *B* moves, while *everything else in the figure adjusts its position accordingly*. In particular, *E* maintains its status as the midpoint of segment *BC*, and *D* holds its relative position on segment *AB*. These adjustments continue until you release the button. This is called *dragging* point *B*. Points *A* and *C* can also be dragged. If you try to drag either *E* or *F*, however, the whole triangle moves rigidly. Think about why this happens. If you try to drag *D*, you will find that it *does* move — but only along the segment it was placed on. Notice that the intersection *G* of the medians *AE* and *BF* is not usually on segment *CD*.

4. Click **Measure** to open a new dialog box. The cursor is blinking in an edit box. Type the ratio *AD/AB* into the box (the program does not distinguish between upper and lower case letters, by the way) and press *Enter*. The current value of this ratio is shown in both the dialog box *and* the drawing window. This is because the measurement dialog *must* be closed (just press *Escape*) before drawing operations can resume. Return to dragging point *D* along *AB*. The displayed value of *AD/AB* tells you the exact position of *D* on the segment. Notice the value of this ratio when *G* seems to be on segment *CD*. Is it what you would expect?

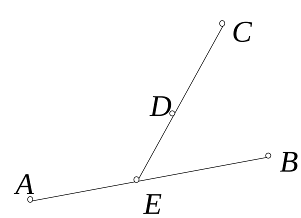
5. Not only does the program assign labels to new points, it also centers the labels directly over the points, and it puts measurements in the upper left corner of the drawing window. You can override these choices. First click **Btns/Text** to put the mouse into a new mode. Point at label *A* and press the *right* button. A small dialog box opens, giving you the opportunity to change the label. Type *P* into the box and press *Enter*. You will see label *A* change to *P*. In the same way, change label *B* to *Q*. Notice that the displayed ratio is now named *PD/PQ*. Next point at any label in the figure, hold down the *left* button, slide the mouse a short distance, and release the button. The point does not move but the label does. Notice that the point is marked by a small circle, usually hidden by the label. Click **View/Labels/Offset** to move all the labels (this seldom places every label satisfactorily, however). Click **View/Labels/Center** to recenter all the labels. To choose a different font for the labels, click **View/Labels/Font**.

6. When the mouse is in **Btns/Text** mode, the right button can also be used to place *new* text (or edit old text) anywhere on the screen, and the left button can be used to drag any text to a new location. For example, drag the measurement to a spot that is closer to segment *PQ*. To send all calculations back to their default positions, click **Other/Home measurements**.

7. Make a fresh start by clicking **Edit/All delete**. When the program asks whether you want to save your work, respond **no**.

8. Put the mouse into **Btns/Segment** mode, right-click four new points, then use the left button to create segments *AB* and *CD*. Put the mouse into **Btns/Drag points** mode, and drag some of the points so that the segments intersect. Here is a new way of labelling the intersection that is occasionally useful: Click **Point/Intersection/Line-line**, type *AB* into one edit box, *CD* into the other, and press *Enter*. Label *E* should appear. Press *Escape* to close the box.

9. Make sure that **Other/Autoextend** does not have a check mark. (If necessary, click it to remove the check.) Drag *B* somewhere so that segments *AB* and *CD* no longer intersect, and watch *E* disappear. Now activate the **Other/Autoextend** feature by clicking it. Notice that the segments are now extended as much as necessary to show their intersection, regardless of where the points are dragged.



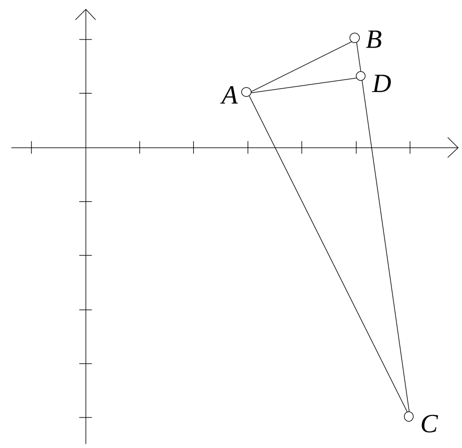
10. Click **Edit/Undo** (or press *Alt+Bksp*). This undoes the most recent construction step, so intersection *E* is no longer labelled. Click **Edit/Undo undo** (or press *Ctrl+Bksp*) to redo the last step. You can always click **Other/Lists/History** to see a step-by-step description of the current figure. This text window closes automatically as soon as you do anything to the drawing. (It can also be closed in the conventional Windows fashion. Clicking *Escape* does not do anything, however, for this is not a dialog box.)

11. Make a fresh start by clicking **Edit/All delete**, then click **Point/Grid** to open a coordinate-entry dialog box. Type 3 into the *x* box, 1 into the *y* box, and press *Enter* (or click **Label**). Point $A = (3, 1)$ should appear. In the same way, label the points $B = (5, 2)$ and $C = (6, -5)$. Press *Escape* to close this dialog box. The coordinate axes disappear at the same time. To keep them in the background permanently, click **View/Grid**.

12. Put the mouse into **Btns/Drag points** mode, then discover that you can not drag *any* of the points without moving the whole sheet of graph paper. This is because the points were defined to keep them in one place.

13. Click open the **Measurement** dialog box. Type <*ABC* into the edit box and press *Enter*. The size of angle *ABC* is displayed. The inequality symbol tells the program that an angle size is requested. If you try to describe the same angle as just <*B*, the program will not respond, for it wants a three-label description. Type *ABC* into the edit box and press *Enter* to see the area of triangle *ABC*. Type *AB*+*BC*+*CA* and press *Enter* to see the perimeter of triangle *ABC*. To deselect the highlighted perimeter in the dialog's list box, click it. Then click the angle measurement to select it. Clicking the **hide** button makes the measurement disappear from the screen, and clicking **show** makes it reappear. Press *Escape* to close the dialog box.

14. If you want dimensional information appended to displayed measurements (to designate length, area, and angles), the menu item **View/Units** must be checked. Until the screen is forced to refresh itself, nothing happens after you change the status of this item.



15. Click **Line/Perpendiculars/Altitude**, type *BC* into the *perp to* box, *A* into the *from pt* box, and press *Enter*. Segment *AD* appears, constructed so that *D* is on segment *BC* and angle *ADB* is a right angle. Press *Escape* to close the dialog box.

16. Click open the **Measurement** dialog box again. Type *D* and press *Enter*. You should see that the coordinates of *D* are (5.1, 1.3). To confirm that *AD* does intersect *BC* perpendicularly, type <*ADB* and press *Enter*. Press *Escape* to close the dialog. Click **Other/Lists/Segments** to open a text window that enumerates all combinations of collinear points in the figure. Because the coordinate axes are showing, a Cartesian equation is displayed for each line, as well as its slope. This window must be closed in conventional Windows fashion.

17. If you would prefer to have the mouse functions more visible (instead of hidden in a menu), click **Btns/Toolbar**. This small dialog box displays the current mouse function, is movable, and gives you another place to alter the mouse function. Put the mouse into **Drag points** mode. As you noticed earlier, dragging any of the labelled points simply slides the sheet of graph paper across the screen. Do so now, and notice that the displayed measurements slide too. They can actually disappear from view. Stop when you have made the measurements disappear. Then click **Other/Home measurements** to relocate them in the upper left corner of the window. Slide the figure again, trying to make some of its vertices disappear from view (if your drawing window does not fill the screen, you can make the entire triangle disappear). To quickly restore any figure to the center of the screen, so that all of its parts are visible, click **View/Window** (or press *Ctrl+W*). This also repositions any measurements.

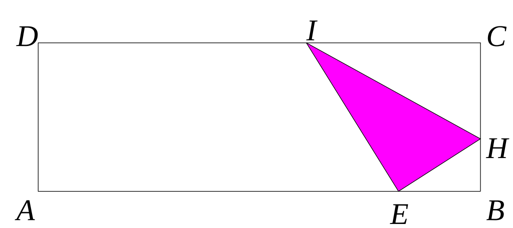
18. Click **Edit/All delete** to begin a new figure. If the coordinate axes are still showing, click **View/Grid** to turn them off. Click **Shape/Polygon/Parallelogram**, leave 12 in the first *side* box, change the *angle* to 90 and the second *side* to 4, then press *Enter* (or click **OK**). A rectangle $ABCD$ should appear. (Because it is three times as wide as it is tall, it might not fit well in the current window, but you can change the size and shape of a window by dragging its border.) Put the mouse into **Btns/Segment** mode and right-click a point E onto the segment AB, closer to B than C is. Then use the left button to draw the segment CE.

19. Click **Line/Perpendiculars/Bisector**, type CE into the edit box, and press *Enter*. The bisector FG goes through the midpoint F of segment CE, and it should also intersect sides BC and CD. Use the right button to label these intersections, with H on BC and I on CD. Then use the left button to draw segments EH and EI.

20. Time to clean up. Points F and G are no longer needed, so click **Edit/Points delete**, type FG into the box, and press *Enter*. Click **Edit/Segment delete**, type the list CE,CH,CI into the box, and press *Enter*. To turn the bisector HI into a segment, click **Line/Extensions**, type the list HI,IH into the box, and press *Enter*. (The rays HI and IH are thereby turned off. Rays can be turned on in the same fashion.)

21. Use the left button to reconnect segments CH and CI. The reason for this strange move will be explained next. Click **View/Highlights/Style lines**, type the list CH,CI into the edit box, click the **dot** button, and press *Enter* (or click **apply**). Because the program was told to *forget* that C, I, and D are collinear (this was the reason for erasing segment CI and then redrawing it), the dotted style does not apply to DI — otherwise the entire side CD of the rectangle would have been dotted, and this was not the plan. Close the dialog box. A final decorative touch: Click **View/Highlights/Fill**, type EHI into the edit box, click the magenta square, and press *Enter*. Triangle EHI should now be filled with color. Close this dialog box, and press *Ctrl+W* to center the drawing, which should now look like the illustration below.

22. This construction is meant to simulate the folding of a rectangular sheet of paper, so that one corner (C) is matched with a point (E) on another edge. The dotted segments mark where the paper used to be, before it was folded over, and the red color is found on the underside of the sheet.



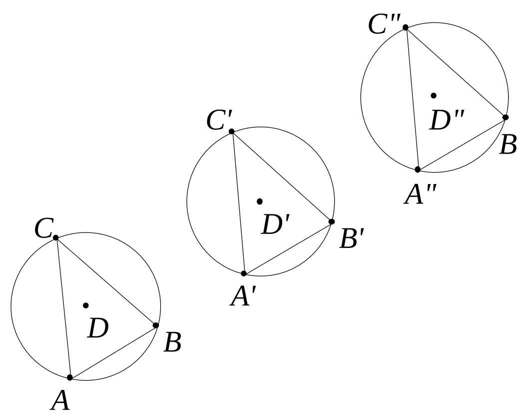
23. Put the mouse into **Btns/drag points** mode and make sure that the **Other/Autoextend** feature is unchecked. Use the left button to slide E along side AB. Move the mouse slowly, keeping E close to B. If you slide E too far, the construction will collapse, for the instructions make sense only as long as H is on side BC and I is on side CD.

24. Open the **Measurement** dialog and ask for $<BHE$ and $2<HIC$ (one at a time, pressing *Enter* after each). *Escape* from the dialog box. Notice that the equality of these two measurements is unaffected by moving E along AB. Explain why this could have been expected.

25. Click **Edit/All delete** to begin a new figure. (It is possible to have several drawings open at the same time — just click **2-Dim** again on the main menu bar — but this is not necessary for this tour.) Click **Shape/Random/Triangle**. This is a one-click method of putting three random points on the screen and connecting them with segments.

26. Click **Circle/Circumcircle**. The resulting dialog box already has *ABC* highlighted in the edit box, so just press *Enter*. The program constructs the circle that goes through all three vertices of the triangle. The center of the circle is labelled *D*. The program keeps track of each circle by recording its center and at least one point on the circle. Deleting the center of a circle would therefore require that the circle itself be deleted. To safeguard against this happening inadvertently, the program will not let you do it: Click **Edit/Point delete**, type *D* into the edit box, and press *Enter*. Click **OK** to close the error-message box. The same message would result if you tried to delete a point that was the only point marked *on* a circle.

27. Click **Transformation/Translate**, type 2 into the *multiple* box, and *AB* into the *vector* box (if it is not already there). Leave the *apply to* box alone, for it lists all the vertices currently in the figure, and that is the plan. To see the result of sliding the whole figure twice the length of vector *AB*, press *Enter*. Notice that the new figure consists of two triangles, two circles, and eight vertices, and that the four new vertices have been labelled by attaching primes to the original labels. Click **Transformation/Again** (or just press *F7*). This applies the current transformation to its most recent images. The figure should now look somewhat like the illustration.



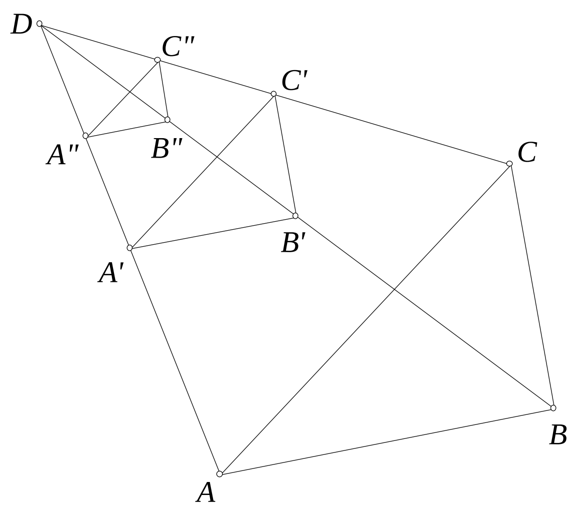
28. With the mouse in **Btns/Drag points** mode, drag vertex *C* around the screen, and notice what happens to all the images. Because the translation is defined in terms of segment *AB*, the effect is quite different if you drag *B* around the screen instead. Try it. Also notice that you can not drag points other than *A*, *B*, or *C* without dragging the entire figure rigidly.

29. Click **Transformation/Translation** again, type 2 into the *multiple* box, type *AC* into the *vector* box, and leave the *apply to* box filled with the default list of all vertices. Press *Enter*, then press *F7*. You should now see nine triangles, nine circles and their centers, and thirty-six labels. The program obtains labels by appending special keyboard characters to upper-case letters. The nine possiblities for each letter are illustrated by the sequence *A*, *A'*, *A''*, *A~*, *A?*, *A%*, *A!*, *A&*, *A`*. Having this many labels on the screen at the same time can be quite confusing. Press *Ctrl+L* (which is easier than clicking **View/Labels/Hide**) to turn all the labels off. (They can be turned on in the same way.) With the labels off, press *Ctrl+D* (which is the same as clicking **View/Labels/Dot mode**) several times. Notice that points are marked by open circles, closed circles, or nothing at all. With the open circles on the screen, click **View/Labels/Bullet size**, type a one-digit number into the box, and press *Enter*. The default size for the circles is 5, but you may prefer a different size (and the program will remember).

30. Click **Edit/All delete** to begin a new figure. Click **Shape/Random/Triangle** again. With the mouse in **Btns/Segment** mode, right-click a random point *D* onto the screen, well away from triangle *ABC*. Click **Transformation/Dilate**. This activates a dialog box that is used to define both rotations and dilations (or combinations of the two). It is now configured for a simple dilation, which means that 0.0 shows in the *angle* box. Type *D* into the *center* box, *ABC* into the *apply to* box, and press *Enter*. Press *F7* once to apply the dilation again.

31. Use the left button to draw segments *DA*, *DB*, and *DC*. Points *A′* and *A″* will look like they also lie on segment *DA*. Because this property is indeed built into the definition of a dilation, you know that these points *are* collinear. Nevertheless, the points were not marked *on* the segment (which appeared later in the construction, anyway), and the program does not know any theorems of geometry, so it does not recognize that *A′* and *A″* are in fact on *DA*. To see evidence of this, click **Edit/Segment delete**, type *AA′* into the box, and press *Enter*. Click **OK** to close the error-message box.

32. The figure should resemble the illustration. Put the mouse into **Btns/Drag points** mode, and drag the primary vertices (namely *A*, *B*, *C*, and *D*) around the screen. The response of the figure when *D* is moved is quite different from the response of the figure when *A* is moved.



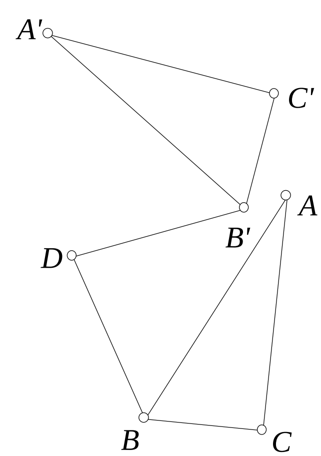
33. If the **Transformation/Label saver** item is checked, click this item to remove the check. Then click **Transformation/Translation**, type *AB* into the *vector* box, *ABC* into the *apply to* box, and press *Enter*. A new triangle appears, probably named *A~B~C~*. According to the definition of vector translation, points *A~* and *B* have to coincide. Nevertheless, *Wingeom* has assigned a new label to what is really the same point. The program thinks that the points are different — indeed, because *A~* had to be *calculated*, it might actually differ from *B* in the twentieth decimal place! This is a situation when you might feel compelled to override the program's lack of understanding, and you *can*: Click **Edit/Undo** (or press *Alt+Bksp*) to undo the translation, click **Transformation/Label saver** to turn this feature back on, then click **Edit/Undo undo** (or press *Ctrl+Bksp*) to redo the translation. Notice that point *A~* does not appear this time. The program has been given permission to regard the nearness of *A~* and *B* as sufficient reason to *identify* these points in its records. This feature only applies to new points generated by the **Transformation** menu, by the way.

34. Click **Edit/All delete** to begin a new figure. Click **Shape/Random/Right triangle** (notice the word *right*). Even though the vertices of this figure are randomly generated, they can *not* be dragged independently of each other. Try it — the triangle moves rigidly as a unit. This is because the program has been taught to respect the right-angle definition.

35. Put the mouse into **Btns/Segment** mode and right-click a random point *D* onto the screen, somewhere outside triangle *ABC*. Click **Transformation/Rotation**, type *D* into the *center* box, notice that the *dilation factor* is 1 (meaning no dilation), type *ABC* into the ap*ply to* box, and type 90# into the *angle* box (do not leave out the symbol #). Press *Enter* to see the result of rotating triangle *ABC* around the pivot point *D*. The size of the rotation angle depends on the value of the number #. Before discovering its value in the next paragraph, estimate it by examining your figure.

36. Click **Animate/# slide**. The resulting dialog box (which can be left open indefinitely) displays the current value of #, and gives you a few ways to change that value. One method is to type a new value for the highlighted text, so type 0.5 into the edit box and press *Enter*. Notice that the figure changes when the value of # does: You now see the result of applying a 45-degree rotation, centered at *D*, to triangle *ABC*. Point at the arrow at one end of the scroll bar, press the left button, and hold it down. The control will slide slowly, the value of # changing as a result. As # changes, so does the size 90# of the rotation angle. You can also slide the control directly by dragging it. The simplest thing is to click **autoreverse**, which moves the bar automatically back and forth. Notice that the dialog box disappears, and that the caption on the drawing window tells you that you must press the *Q* key if you want this animation to stop. Watch for a while, then press *Q*.

37. Click **Line/Segments**, type *BDB'* into the edit box, and press *Enter*. The segments *BD* and *DB'* appear. Click **View/Highlights/Style line**, type *BDB'* into the edit box, click the **dot** button, and press *Enter*. Press *Escape* to close this dialog box. Your figure should now resemble the illustration.



38. Click open the **Measurement** dialog. Type # and press *Enter*. Type 90# and press *Enter*. Type <*BDB'* and press *Enter*. Close the dialog. Explain the coincidence of displayed values. Return to varying the values of #, and notice the changing screen display.

39. Click **Transformation/Rotation**, type *D* into the *center* box, type *A'B'C'* into the *apply to* box, check that the *angle size* box still shows 90#, then press *Enter*. Triangle *A'B'C'* appears. Press *F7* four times. There should be seven triangles on the screen, the last being *A!B!C!*.

40. Check the feature **Transformation/Label saver**. Then type 2/3 into the edit box that displays the current value of #, and press *Enter*. Explain why there are now only six triangles on the screen. Where did *A!B!C!* go? If you need a hint, type 0.68 into the edit box and press *Enter*.

41. To change the number of decimal places in displayed calculations, click **Edit/Decimals**, type a nonnegative integer less than 19, and press *Enter*. The new format will become apparent as soon as any numerical display is refreshed. For example, you could use the mouse to drag the original triangle *ABC* around the screen.

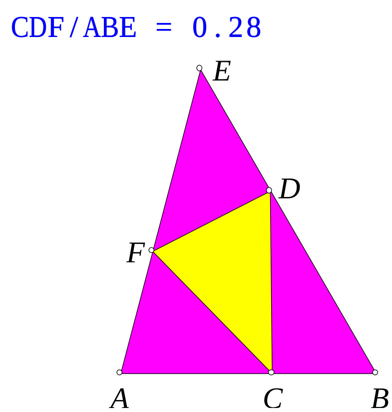
42. Click **Edit/All delete** to begin a new figure. For a very simple start, click **Shape/Segment**, and press *Enter*. A segment of unit length (this refers to the scale on the underlying graph paper) appears. Click **Line/Angles/New**, type *AB* into the *initial ray* box, 75 into the *angle size* box, and press *Enter*. Ray *AC* appears. Now type *BA* into the *initial ray* box and –60 into the *angle size* box, and press *Enter* again. Ray *BD* appears. Notice the use of a negative sign to describe the second angle. What positive size would have produced the same ray *BD*?

43. The intersection of these rays may well be outside the drawing window. One way to label this (perhaps invisible) point is to click **Point/Intersection/Line-line**, type *AC* and *BD* into the boxes, and press *Enter*. The intersection *E* should now be visible.

44. Triangle *ABE* has been constructed so that its angles are 75, 60, and 45 degrees. To finish the job, click **Edit/Point delete**, type *CD* into the box, and press *Enter*. Click **Line/Extensions**, type the list *AE*,*BE* into the box, and press *Enter*.

45. Type 0.6 into the edit box that displays the current value of #, and press *Enter*. Then click the **Point/Segment division** item. This provides an alternate way of marking points on segments, one that offers unlimited possiblilities for animation and control. For example, type the list *AB*,*BE*,*EA* into the *segments* box. If you were to press *Enter* now, with 0.5 in the *coordinate* box, you would see all three midpoints of the sides of triangle *ABE* appear. Instead, replace the 0.5 by the symbol #, which you know stands for the value 0.6, and press *Enter*. Point *C* appears on segment *AB*, 60% of the way from *A* to *B*, point *D* appears on segment *BE*, 60% of the way from *B* to *E*, and point *F* appears on segment *EA*, 60% of the way from *E* to *A*.

46. Put the mouse into **Btns/Segment** mode and use the left button to connect the vertices of triangle *CDF*. Notice that the program has reused the discarded labels *C* and *D*, by the way. Click **View/Highlights/Fill**, type *ABE* into the edit box, and click **fill** (or press *Enter*). The large triangle now has a red interior. Type *CDF* into the edit box, left-click the yellow box in the fill color dialog, and click **fill** again. After noticing why it was necessary to color the large triangle *first*, close the dialog box. The figure should look like the illustration.



47. Click open the **Measurement** dialog, ask for the ratio of areas *CDF*/*ABE* (remember that it is all right to use lower cases), and close the dialog. The display should show that the area of triangle *CDF* is 28% of the area of triangle *ABE*.

48. Predict the value of the area ratio when *C*, *D*, and *F* are the midpoints of their respective sides. Set # equal to 0.5 to see whether you were right. Can *CDF* be exactly 50% of *ABE*?

49. Make the drawing window the active window (click its title, for example), press *Ctrl+L* to turn off the labels, press *Ctrl+D* a couple of times to turn off the circles, then click **Autoreverse** in the parameter dialog. Remember to press the *Q* key to stop the animation.

Parabola Lab

1. To construct a parabola, you need a directrix and a focus. Put the mouse into **Btns/Segment** mode and right-click three points onto the drawing surface, with *A* near the bottom on the left, *B* near the bottom on the right, and *C* near the center of the window. Exact placement is not important, for adjustments will be made later. Now use the left button to connect *A* to *B*: Point at either vertex, hold down the left button, drag the pointer to the other vertex, and release. Segment *AB* appears.

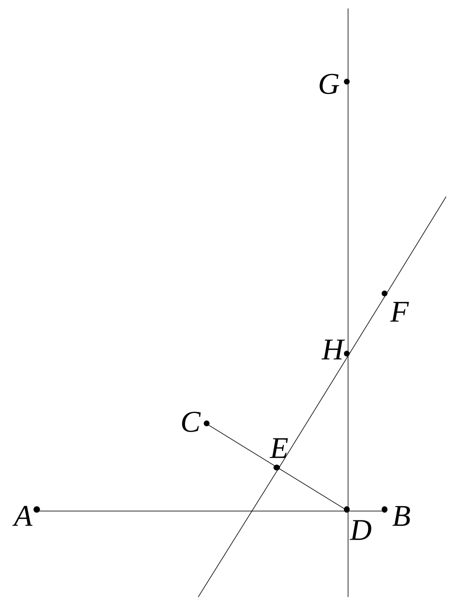
2. Right-click a random point *D* onto segment *AB*. To check that it really *is* on the segment, put the mouse into **Btns/Drag points** mode and try to drag *D*. It should only slide along *AB*.

3. Put the mouse back into **Btns/Segment** mode and use the left button to connect *C* to *D*.

4. To draw the perpendicular bisector of segment *CD*, click **Line/Perpendiculars/Bisector**, type *CD* into the edit box, and press *Enter* (or click **OK**). The program first marks *E* at the midpoint of *CD*, then draws perpendicular line *EF*. (For inventory purposes, the program needs at least two points on every line, so it introduces *F* as well.) All the points on line *EF* have a special property — what is it?

5. Click **Line/Perpendiculars/General** to draw the line that is perpendicular at *D* to segment *AB*. Type *AB* into the *perpendicular to* box (it might already be there) and *D* into the *through point* box. Press *Enter* (or click **Mark**) to see the line *DG*. The view frame will probably be repositioned because of these new points. Press *Escape* to close the dialog box.

6. The simplest way to label the intersection *H* of lines *DG* and *EF* is to point at it and click the right button. (If the intersection does not lie within the window, this method will not work, however. Another way is to click **Point/Intersection/Line-line**, type *DG* and *EF* into the two edit boxes, and press *Enter*.) The figure should now resemble the illustration.



7. Put the mouse into **Btns/Drag points** mode. Then use the left button to drag *D* back and forth along segment *AB*. If the menu item **Other/Autoextend** does not have a check mark, give it one by clicking it. This feature enables you to slide *D* past the ends of segment *AB*.

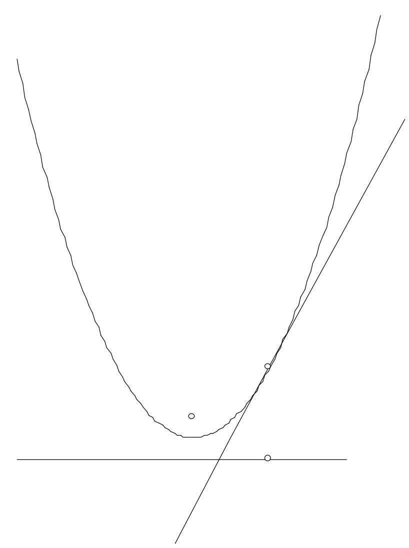
8. No matter what position *D* has on line *AB*, it is always the point on *AB* that is ____________ to *H*. No matter what position *D* has on line *AB*, what can be said about the distances from *H* to the focal point *C* and from *H* to the directrix *AB*?

9. Now that point *H* has been constructed, points *F* and *G* are no longer needed, so click **Edit/Point delete**, type *FG* into the edit box, and press *Enter*. Lines *DH* and *CD* have also served their purposes, so click **Edit/Segment delete**, type the list *DH,CE,ED* into the edit box, and press *Enter*. (The reason for typing *CE,ED* instead of *CD* is that the program will not let you delete a segment that contains an interior point.) Click **View/Window** (or just press *Ctrl+W*) to reposition the figure in the window.

10. As you slide *D* along *AB*, try to visualize the path that *H* is following. Here is an easy way of making this path appear: Click **Animate/Temporary trace**, type *H* into the edit box, and press *Enter*. When you slide *D* along *AB* now, the program retains all of the corresponding positions for *H* in the figure. It is better to move *D* slowly. This is only a temporary display (it disappears as soon as the screen is refreshed), which is convenient for certain explorations.

11. What would the path look like if *C* were closer to *AB*? What would the path look like if *C* were further from *AB*? To explore such questions, first drag *C* to a new position and then regenerate the path by sliding *D* along *AB* again.

12. For a smooth, permanent tracing, click **Animate/Tracing/new**. Click the **vertex** button and type *D* into the adjacent edit box. This tells the program that the tracing is generated by dragging vertex *D*. It is not necessary to change the *low* or *high* boxes, but do type 100 into the *steps* box. Finally, type *H* into the *pen on* box and press *Enter* (or click **OK**). The program plots *H* for 100 positions of *D*, and connects them.



13. Press *Ctrl+L* (which is easier than clicking **View/Labels/Hide**) to turn off the labels. They are no longer needed. If the points are not marked by circles, press *Ctrl+D* (which is easier than clicking **View/Labels/Dot mode**). To hide unnecessary points, click **View/Labels/Individual**, type *ABE* into the edit box, uncheck the **Label** box, and press *Enter*. Only the three points *C*, *D*, and *H* should now be visible, each identified by a circle. The figure should now look like the illustration.

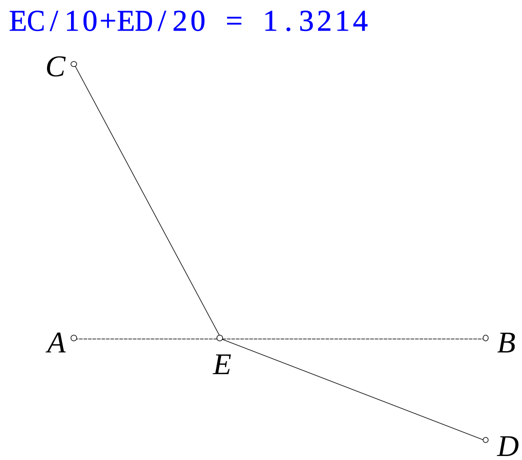
14. The perpendicular bisector *EH* has been left in the figure for a reason — it bears a special relationship to the parabola traced by *H*. As you slide *D* along *AB*, think of some words to describe this relationship: ____

15. Unless the menu item **Animate/Monitor tracings** is checked, the tracing will not respond when you try to drag points other than *D*. Checking this item is *not* a recommended way to explore, however — the large amount of drawing that must be done as the mouse is dragged will make the program seem very sluggish. It is better to simply click **Animate/Retrace** when necessary.

Snell's Law Lab

1. Click **Point/Grid**. A coordinate grid appears, and a dialog box that shows the values $x = 0$ and $y = 0$. Press *Enter* (or click **Label**) to put $A = (0, 0)$ onto the screen. Type 12 into the x box and press *Enter* again to make $B = (12, 0)$ appear. Mark similarly the points $C = (0, 8)$ and $D = (12, -3)$. Press *Escape* to close the dialog box. (The coordinate grid disappears, too.) Click **View/Window** (or press *Ctrl+W*) to center the figure within the window.

2. Check that the mouse is in **Btns/Segment** mode, then use the left button to draw the segment *AB*. Use the right button to click a point *E* onto segment *AB*. Then use the left button to draw segments *EC* and *ED*.



3. If the item **View/Units** is checked, click it to turn off the check. Open the **Measurement** dialog box and put the sum *EC*/10 + *ED*/20 on display. You can press *Escape* to close the dialog box. Recall the simple story (item 2 on page 16) that goes with this measurement.

4. Put the mouse into **Btns/Drag points** mode. Use the left button to drag *E* back and forth along segment *AB*, trying to make the displayed measurement as *small* as you can.

5. Once *E* is in its optimal position, open the **Measurement** dialog, ask for the length of *AE*, and press *Escape* to close the dialog box. Record the two values displayed in the figure:

   AE = ________________ *EC*/10 + *ED*/20 = ________________

   Compare this numerical data with the values you obtained when you did this problem on your graphing calculator. Contrast the two methods.

6. If the objective had been to make the value of the sum *EC*/10 + *ED*/10 as small as possible, your search would not have led you to the same point *E*. Guess which point on segment *AB* would have been the optimal choice for *E*, then check your guess by doing the search. Begin by using the **Measurement** dialog to delete *EC*/10 + *ED*/20 and replace it by *EC*/10 + *ED*/10. Record your findings:

   AE = ________________ *EC*/10 + *ED*/10 = ________________

7. New example: Search for the point E that makes $EC/20 + ED/10$ as small as possible.

   $AE =$ ________________ $\qquad$ $EC/20 + ED/10 =$ ________________

   Notice that segment EC is longer than it was in either of the two preceding solutions. Explain why this could have been predicted.

8. Click **Line/Segment**, type the list *AC*,*BD* into the edit box, and press *Enter*. Segments AC and BD should appear. Use the mouse to drag E along segment AB. As E moves from A to B, what happens to the sizes of angles ACE and BDE? Are they ever the same size?

9. You have now explored three problems of the same general form: Find the point E that minimizes the value of the expression $EC/r_C + ED/r_D$. The expression represents *time*, because the numerators of the fractions represent *distances* and the denominators represent *rates*. A few more examples and measurements are needed to reveal a pattern. Fill in the missing entries of the table below, and notice that there are two new quantities of interest. Each row requires a separate search.

r_C	r_D	EA	$EC/r_C + ED/r_D$	EA/EC	EB/ED
10	30				
10	20				
10	10				
40	30				
20	10				
30	10				

10. As the rate r_D decreases, what happens to the optimal position of the point E? What happens to the length of segment ED? What happens to the size of angle BDE? Are r_D and angle BDE linearly related?

11. The simple pattern known as *Snell's Law* relates r_C, r_D, EA/EC, and EB/ED. It can be inferred by examining the table. Express this relationship in words. Express it in symbols.

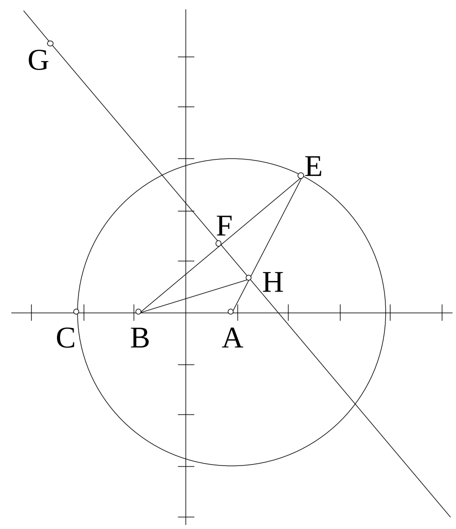
12. Write a trigonometric description of Snell's Law that relates the rates r_C and r_D to the angles ACE and BDE.

Ellipse Lab — Part I

1. Click **Points/Grid**. Type 9 into the *x*-box and press *Enter* (or click **Label**). Point $A = (9, 0)$ should appear on the screen. In a similar fashion, mark the points $B = (–9,0)$ and $C = (–21,0)$. Press *Escape* to close the dialog box. (The coordinate axes disappear as well. If you want to keep them on the screen, click **View/Grid**.)

2. Click **Circle/Radius-center**, type *A* into the *center* box, *AC* into the *radius* box, and press *Enter* (or click **draw**). You can also press *Escape* to close the dialog box, which is no longer needed. You should now see the circle centered at *A* that goes through *C*.

3. Put the mouse into **Btns/Segment** mode. Point anywhere *on* the circle and click the right button to mark *E*. Then use the left button to draw the segments *AE* and *BE*.



4. The point *E* is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns/Drag points** mode, point at *E*, press the left button, and hold it down while you slide the mouse.

5. Click **Line/Perpendiculars/Bisector**, type *BE* into the edit box, and press *Enter*. Notice that the program marked midpoint *F* on segment *BE* when it drew the bisector *FG*.

6. Click **Point/Intersection/Line-line**, type *FG* into one box and *AE* into the other, and press *Enter* (or click **mark**). Label *H* should now mark the intersection of line *FG* and radius *AE*. Press *Escape* to close the dialog box.

7. Click **Line/Segments**, type *HB* into the edit box, and press *Enter*. The figure should now resemble the illustration.

8. As you answer the following questions, you may wish to move *E* around the circle (the mouse is probably still in **Btns/Drag points** mode) and observe what happens to the constructions, especially point *H*. The **Measurement** dialog box may also be useful.

   (a) How do the lengths *HB* and *HE* compare?

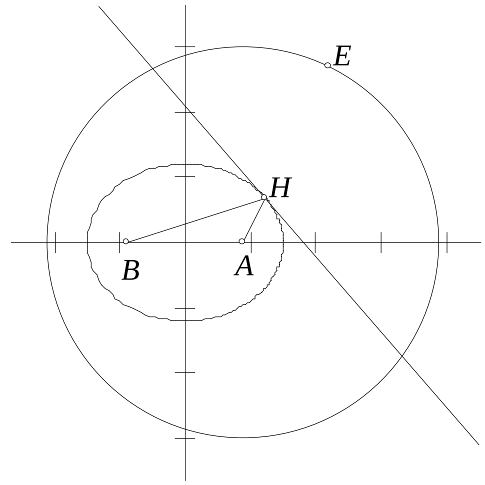
   (b) What must always be true about the sum of the distances *HB* and *HA*, no matter what the position of point *E*? Explain.

   (c) Describe what the trace of point *H* will look like as point *E* moves around the circle.

Ellipse Lab — Part I (continued)

9. To see a picture of the path of *H*, click **Animate/Tracing/new**. Click the **vertex** box and type *E* into the adjacent box (this tells the program to produce the tracing by sliding *E*), type 100 into the *steps* box, *H* into the *pen on* box, and press *Enter*. The program now plots 100 different positions for *H*, connects them with a curve, and superimposes the construction.

10. It is time to clean up a bit. Click **Edit/Segment delete**, type *EH* into the box, and press *Enter*. Click **View/Labels/Individual**, type *CFG* into the edit box, click the *label* box to uncheck it, press *Enter*, then *Escape* from the dialog. Click **View/Highlight/Color lines**, type *EB* into the edit box, click the **invisible** button, and press *Enter*. Then type the list *BH*,*AH* into the same edit box, click the **blue** button, and press *Enter* again. Press *Escape* to close this dialog. Your figure should now resemble the illustration.



11. Click **Measurement**, type *H* into the edit box, and press *Enter*. The coordinates of point *H* are added to the figure. Close this dialog box (press *Escape*) and return to sliding *E* around the circle. This makes *H* slide along a curve known as an ________________. The path of *H* intersects the coordinate axes at four lattice points. Find these coordinates exactly. Is it a coincidence that the distance between the *x*-intercepts is equal to the radius of the circle? Explain.

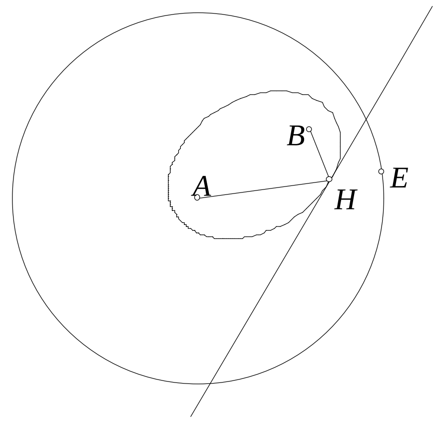
12. Summarize the construction roles played by point *A*, point *E*, radius *AC*, and line *FG*.

13. Problem 2 on page 47 implies that the focal radius *HA* is always three fifths of the distance from *H* to the vertical line $x = 25$. Here is one way to confirm this fact: Use the **Point/Grid** dialog to plot two points on the line, say $I = (25, -30)$ and $J = (25, 30)$. Draw the line by clicking **Line/Line**, typing *IJ* into the edit box and pressing *Enter*. Then mark the point *K* on *IJ* that is closest to *H* by clicking **Line/Perpendiculars/Altitude**, typing *IJ* into the *perp to* box, *H* into the *from point* box, and pressing *Enter*. Now open the **Measurement** dialog and ask for the values of *HA*, *HK*, and *HA/HK*, one at a time. Close this dialog and return to sliding *E* around the circle, keeping an eye on the displayed values. Notice that the ratio *HA/HK* does indeed have a constant value, even though *HA* and *HK* are *not* constant.

14. The technical name for the line $x = 25$ is ____________________. What is the other line that plays the same role as $x = 25$ does?

15. The technical name for the (constant) ratio *HA/HK* is ____________________. Verify that the ratio *AB*/*AE* is also 3/5. This is not a coincidence.

Ellipse Lab — Part II



16. Click **Edit/All delete** to start a new sketch. Instead of plotting three specific points as in Part I, put the mouse into **Btns/Segment** mode, and use the right button to mark three random points *A*, *B*, and *C*. Click **View/Grid** and remove the the axes. You should now repeat the same constructions as in steps 2 through 8 above. Pause after you construct the circle centered at *A*, to make sure that point *B* is inside the circle. If it is not, simply drag it inside before proceeding. **File/Save** the figure when you have finished.

    Unlike the specific points in Part I, these points *A*, *B*, and *C* can be dragged around the screen. It is interesting to study the effect that different positions have on the appearance of the elliptical path of point *H*. Two possible approaches:

17. Click **Animate/Temporary trace**, type *H* into the edit box, and press *Enter*. Watch what happens when you slide *E* around the circle. The elliptical path of *H* appears, in a rough and *temporary* form — the collection of points persists on the screen until you do something to the figure. Drag *B* close to the center *A* (the temporary path disappears), then slide *E* around the circle again to obtain another temporary path. Notice that the shape of the ellipse is different this time. Now drag *B* close to the circle and repeat, noticing the new shape. You can also experiment with changing the size of the circle, by dragging *C* before you slide *E*. Keep in mind that the sum *HA*+*HB* always equals the ______________ of the circle. The sum *HA*+*HB* is what dimension of the ellipse itself? (see step 11 of Part I)

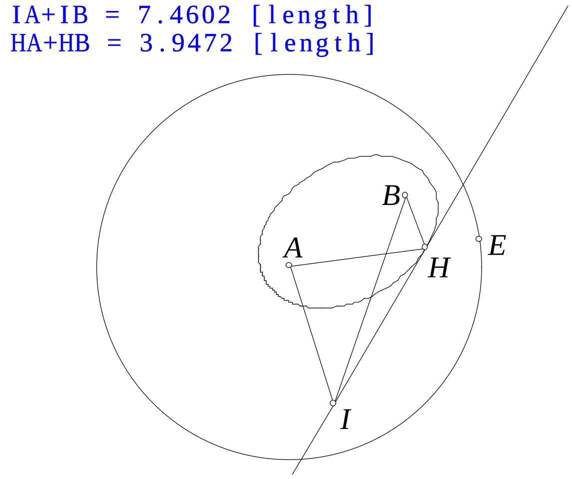
18. Click **Animate/Tracing/new** and repeat step 9 of Part I. Unless the menu item **Animate/Monitor tracings** is checked, this tracing will not respond when you try to drag points other than *E*. Checking this item is *not* a recommended way to explore, however, as you will see when you try it. The large amount of drawing that must be done as *B* is dragged will make the program seem *very* sluggish. It is better to leave this item unchecked, therefore, and simply click **Animate/Retrace** (or press *Ctrl+X*) to refresh the ellipse after moving *B*.

19. The ratio *AB*/*AE* is called the *eccentricity* of the ellipse. It describes the shape of the ellipse. In the Part I example, the eccentricity was 3/5. Use the **Measurement** dialog to display the value of *AB*/*AE*. When you move points *A*, *B*, or *C*, the eccentricity will change. What is the *range of values* of the eccentricity? What words would you use to describe the shape of the ellipse when the eccentricity has (a) a value near zero? (b) a value near one?

20. <u>A Challenge</u>**:** In Part I, the line $x = 25$ was a directrix for the ellipse (and $x = -25$ was the other directrix). Recall that, for *any* point *H* on that ellipse, the distance from *H* to the focus *C* was three fifths of the distance from *H* to the corresponding directrix. This should help you construct the directrices for the generic ellipse in your figure. These lines should of course change whenever you change the ellipse by moving *A*, *B*, or *C*.

Ellipse Lab — Part III

21. Use **File/Old** to retrieve the figure you saved in step 16 above. Click **Animate/New tracing** and repeat step 9 of Part I, so that the elliptical path of *H* is on the screen. With the mouse in **Btns/Segment** mode, right-click a new point *I* onto the line *FG* (the only line visible in the figure). Use the left button to draw segments *IA* and *IB*. Click open the **Measurement** dialog, ask for the values of *HA*+*HB* and *IA*+*IB* (one at a time), then *Escape* from the dialog.



22. Put the mouse back into **Btns/Drag points** mode and slide *E* around the circle, thereby driving *H* around the ellipse. The displayed measurements should confirm that *HA*+*HB* is constant for all points *H* on the ellipse. This constant is equal to the ___________ of the circle. Now slide *I* along the line *FG*. Notice that *IA*+*IB*, which is evidently not constant, is never ___________ than the sum *HA*+*HB*. What does this tell us about the relationship between the ellipse and the line?

23. Press *Ctrl*+*L* to turn on all the vertex labels. Slide point *I* so that *H* is between *G* and *I*, then **measure** the angles *GHB* and *IHA*. Slide *E* around the circle so that *H* slides around the ellipse, and watch the angle measurements. What important property of the ellipse does this illustrate? Explain how this property ensures that the minimum value of *IA*+*IB* for a point *I* on the line *GH* must indeed occur when *I* coincides with *H*.

Ellipse Lab — Part IV (summary)

24. Click **Edit/All delete** to make a fresh start. With the mouse in **Btns/Segment** mode, right-click three points *A*, *B*, and *C* onto the screen, then use the left button to draw segments *AC* and *BC*.

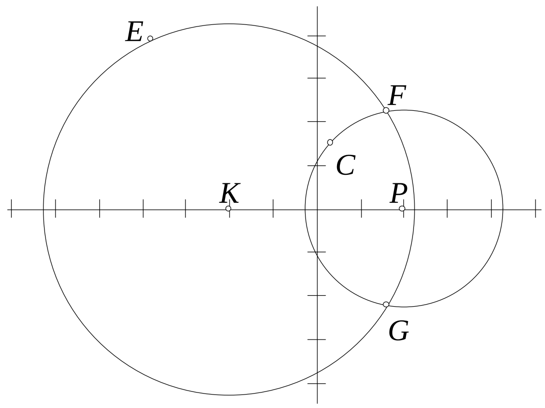
25. Suppose that points *A* and *B* represent the focal points of an ellipse, and that *C* is a point on that ellipse. The problem is to construct the line that is tangent to the ellipse at *C*. This can be done in just two steps. First click **Line/Angles/Bisect**, type *ACB* into the edit box, and press *Enter*. Bisecting ray *CD* should appear. Next click **Line/Perpendiculars/General**, type *CD* into the *perpendicular to* box, type *C* into the *through point* box, and press *Enter*. Tangent line *CE* appears.

26. Because there is no longer a need for the bisecting ray, click **Edit/Point delete** to remove point *D*. Now put the mouse into **Btns/Drag points** mode. Drag *C* around the screen and notice how the line *CE* reacts. Because the sum of lengths *AC*+*BC* is not kept constant, each line *CE* is tangent to a different ellipse. To see the ellipses, click **Shape/Conic/Ellipse**, type *ABC* into the edit box, and press *Enter*.

Hyperbola Lab — Part I

1. Click **Point/Grid**. A coordinate grid appears, and a dialog box that shows the values $x = 0$ and $y = 0$. Type –2 into the x box and press *Enter* to make $A = (–2, 0)$ appear on the screen. Type 2 into the x box and press *Enter* again to make $B = (2, 0)$ appear. Press *Escape* to close the dialog box. The coordinate grid disappears, too. Click **View/Grid** and restore the axes.

2. With the mouse in **Btns/Segment** mode, right-click a random point C onto the screen approximately midway between A and B.



3. Click **Circle/Radius-center**, type B into the *center* box, type BC into the *radius* box, and press *Enter*. Then type A into the *center* box, type $2+BC$ into the *radius* box, and press *Enter* again. Press *Escape* to close the dialog box.

4. The circles you just drew should intersect in two places. Right-click either point. The labels F and G will be applied to the intersections.

5. Put the mouse into **Btns/Text** mode and right-click point A. As invited to by the dialog box, change the label to K. Then right-click point B, and change this label to P. The figure should now resemble the illustration above. This construction is meant to correspond to the text of problem 1 on page 70. Explain the label change for P and K. Explain the choice of $2+BC$ for the radius of the second circle. Explain the significance of points F and G.

6. The coordinates of F and G can be displayed: Open the **Measurement** dialog box, type F and press *Enter*, type G and press *Enter*, then press *Escape*.

7. Put the mouse into **Btns/Drag points** mode. Drag point C (which is actually the only point that *can* be dragged). You will see the coordinates for many points that correctly answer problem 1 on page 70. (It may happen occasionally that the two circles do not intersect, in which case F and G do not appear in the figure, and the displayed coordinates are therefore meaningless.) The y-coordinate of one of these intersection points is 5; its x-coordinate is approximately _____________. Another special case occurs when the intersection points *merge*; the resulting x-coordinate is _____________. Finally, notice what happens when C is dragged to the left of K. Explain why the resulting points F and G do *not* answer problem 3.

8. Click **Animate/Temporary trace**, type FG into the edit box, and press *Enter*. When you drag C around the screen now, the program accumulates all of the corresponding positions for F and G in the figure. More points are plotted when you move C slowly. This is only a temporary display, however — it disappears as soon as the screen is refreshed. A more permanent display is obtained next.

Hyperbola Lab — Part II

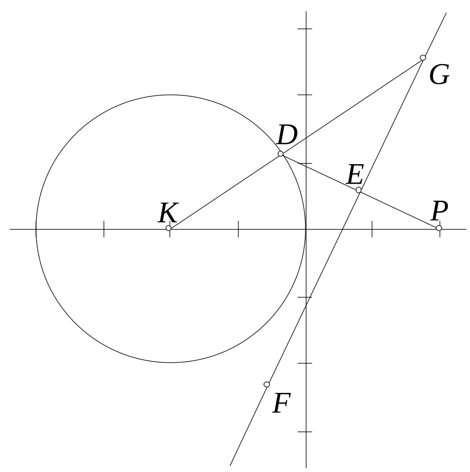
9. Begin a fresh sketch with the same points *A* and *B*. One way to do this is to click **Edit/Undo** (or press *Alt+Bksp*) six times. Another is to click **Edit/All delete** and re-enter the coordinates for *A* = (–2, 0) and *B* = (2, 0) a second time, using the **Point/Grid** menu.

10. Click **Circle/Radius-center**, type *A* into the *center* box, type 2 into the *radius* box, and press *Enter*. A circle of radius 2 appears, with a random point *C* marked on it. Press *Escape* to close the dialog box.

11. With the mouse in **Btns/Segment** mode, right-click a random point *D* onto the circle, roughly midway between *A* and *B*, then use the left button to make the connections *BD* and *AD*.

12. Click **Line/Perpendiculars/Bisector**, type *BD* into the edit box, and press *Enter*. Notice that the program automatically marks the midpoint *E* of segment *BD* when it draws the bisector.

13. Make radius *AD* into a ray by clicking **Line/Extension**, typing *AD* into the edit box, and pressing *Enter*. This is to enable the label to appear.



14. Click **Point/Intersection/Line-line**, type *AD* into one edit box, *EF* into the other, and press *Enter*. You should see label *G* appear at the intersection of ray *AD* and line *EF*. Press *Escape* to close the dialog box.

15. As above, put the mouse into **Btns/Text** mode and change labels *A* and *B* to *K* and *P*, respectively. The figure should now look like the illustration at right.

16. Explain why *GK* – *GP* = *KD*. Explain also why *KD* is 2, no matter what the position of *D*.

17. Click **Animate/Temporary trace**, type *G* into the edit box, and press *Enter*. Put the mouse into **Btns/Drag points** mode and drag *D* slowly around the circle. Explain why the trace of *G* consists of points that already appeared in step 7 above. Notice also that, for some positions of *D*, there is no point *G*.

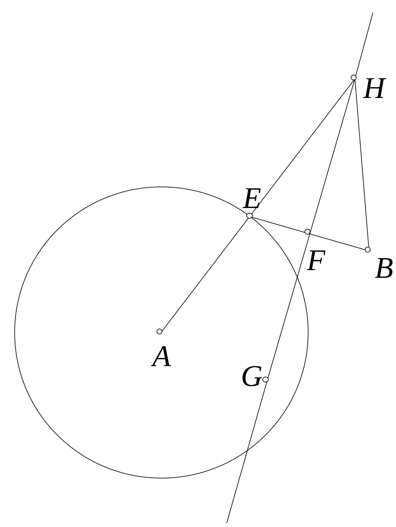
18. For a smooth, permanent tracing, click **Animate/Tracing** to open an inventory dialog, then click **new**. Click the **vertex** button and type *D* into the adjacent edit box. This tells the program that the tracing is generated by dragging vertex *D*. Do not change the *low* or *high* boxes, but do type 100 into the *steps* box. Finally, type *G* into the *pen on* box and press *Enter* (or click **OK**). The program plots *G* for 100 positions of *D*, and connects them.

Hyperbola Lab — Part III

19. Click **Edit/All delete** to start a new sketch, and put the mouse into **Btns/Segment** mode. If the coordinate axes are still on the screen, click **View/Grid** and remove them. Use the right button to mark two random points *A* and *B*, and then a third random point *C* that is closer to *A* than *B* is.

20. Click **Circle/Radius-center**, type *A* into the *center* box, *AC* into the *radius* box, and press *Enter* (or click **draw**). You can also press *Escape* to close the dialog box, which is no longer needed. You should now see the circle centered at *A* that goes through *C*.

21. Point anywhere *on* the circle and click the right button to mark *E*. Then use the left button to draw the segments *AE* and *BE*.



22. The point *E* is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns/Drag points** mode, point at *E*, press the left button, and hold it down while you slide the mouse. When you are done, leave *E* somewhere between *A* and *B*.

23. Click **Line/Perpendiculars/Bisector**, type *BE* into the edit box, and press *Enter*. Notice that the program marked midpoint *F* on segment *BE* when it drew the bisector *FG*.

24. So that the next label will appear, check the item **Other/Autoextend** by clicking it. Click **Point/Intersection/Line-line**, type *FG* into one box and *AE* into the other, and press *Enter* (or click **mark**). Label *H* should now mark the intersection of line *FG* and line *AE*. Press *Escape* to close the dialog box.

25. Click **Line/Segments**, type *HB* into the edit box, and press *Enter*. The figure should now resemble the illustration.

26. As you answer the following questions, you may wish to move *E* around the circle (the mouse is probably still in **Btns/Drag points** mode) and observe what happens to the constructions, especially point *H*. The **Measurement** dialog box may also be useful.

    (a) How do the lengths *HB* and *HE* compare?

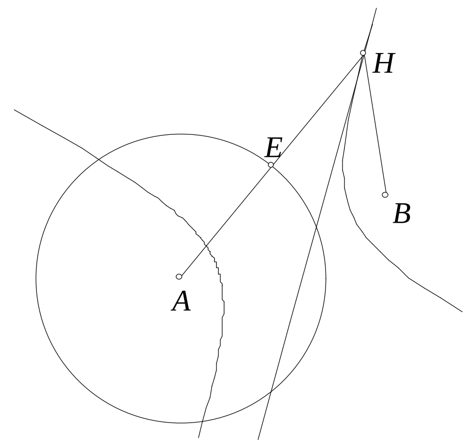
    (b) What must always be true about the difference *HA* – *HB*, no matter what the position of point *E*? Respond carefully — this difference is not always positive.

    (c) Describe what the trace of point *H* will look like as point *E* moves around the circle.

Hyperbola Lab — Part III (continued)

27. To see a picture of the path of *H*, request **Animate/Tracing/new**. Click the **vertex** box and type *E* into the adjacent box (this tells the program to produce the tracing by sliding *E*), type 100 into the *steps* box, *H* into the *pen on* box, and press *Enter*. The program now plots 100 different positions for *H*, connects them with a curve called a *hyperbola*, and superimposes the construction.

28. It is time to clean up a bit. To hide the labels *F* and *G*, click **View/Labels/Individual**, type *FG* into the edit box, click the *label* box to uncheck it, press *Enter*, then *Escape* from the dialog. To hide the segment *EB*, click **View/Highlight/Color lines**, type *EB* into the edit box, click the **invisible** button, and press *Enter*. Then type the list *BH*,*AH* into the same edit box, click the **blue** button, and press *Enter* again. Press *Escape* to close this dialog. Your figure should now resemble the illustration.



29. With the mouse in **Btns/Drag points** mode, you can drag *A*, *B*, and *C* around the screen and see the effect that this has on the tracing. Unless the item **Animate/Monitor tracings** is checked (not recommended), the tracing does not respond to dragging, so you will have to click **Animate/Retrace** now and then. It is interesting to see what happens to the trace when *B* is closer to *A* than *C* is — the hyperbola turns into an ellipse!

30. Summarize the construction roles played by point *A*, point *E*, radius *AC*, and line *FG*.

Hyperbola Lab — Part IV (summary)

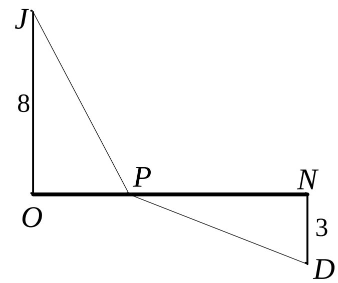
31. Click **Edit/All delete** to make a fresh start. With the mouse in **Btns/Segment** mode, right-click three points *A*, *B*, and *C* onto the screen, then use the left button to draw segments *AC* and *BC*.

32. Suppose that points *A* and *B* represent the focal points of a hyperbola, and that *C* is a point on that hyperbola. The problem is to construct the line that is tangent to the hyperbola at *C*. This can be done very quickly. First click **Line/Angles/Bisect old**, type *ACB* into the edit box, and press *Enter*. Bisecting ray *CD* should appear. To make it into a line, click **Line/Extensions**, type *DC* into the edit box, and press *Enter*.

33. Now put the mouse into **Btns/Drag points** mode. Drag *C* around the screen and notice how the line *CD* reacts. Because the difference of lengths *AC*–*BC* is not kept constant, each line *CD* is tangent to a different hyperbola. To see the hyperbolas, click **Shape/Conic/Hyperbola**, type *ABC* into the edit box, and press *Enter*.

Problems for Snell's Law Lab:

1. Jules is at the point $J = (0,8)$ offshore, needing to reach the destination $D = (12,-3)$ on land as quickly as possible. The shore of this lake is the x-axis, with $O = (0,0)$ and $N = (12,0)$. Jules is in a boat that moves at 10 uph, with a motor bike on board that will move 20 uph once the boat reaches land. The problem is to find the landing point $P = (x,0)$ that will minimize the total travel time from J to D. Assume that the trip from P to D is along a straight line, as shown.



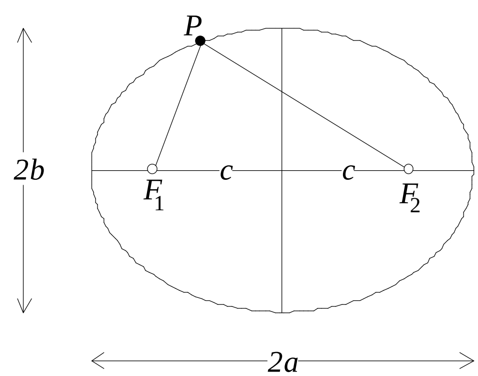
2. (Continuation) Calculate the sine of angle PJO and the sine of angle PDN. These two sine values, together with the two given speeds, fit a simple relationship known as *Snell's Law*, or the *Law of Refraction*. Try to predict what you would find if the boat's speed were increased to 15 uph. To validate your prediction, you will need to solve the problem with the new speed, of course. Write a general statement of this principle.

Problem for Ellipse Lab – Part I:

3. Let $F = (9, 0)$ and choose a point P that fits the equation $16x^2 + 25y^2 = 3600$. Confirm that the distance from P to F is exactly three fifths the distance from P to the vertical line $x = 25$. Repeat this verification for two more points P that fit the equation. Calculate a and c for this ellipse, to show that its eccentricity is 3/5.

Problems for Ellipse Lab – Part II:

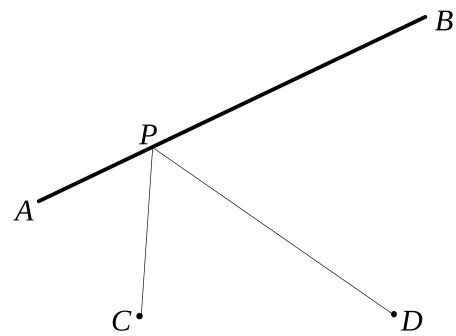
4. *Some ellipse terminology*: For reasons that will eventually become clear, the anchor points F_1 and F_2 for the string are called *focal points* (or *foci*). The focal points are located on the *major* symmetry axis, with the ellipse *center* midway between them. The *vertices* are the points where the ellipse intersects the major axis. As shown, it is customary to let $2a$ be the distance between the vertices, $2c$ be the distance between the foci, and $2b$ be the distance between the intersections of the ellipse with the *minor* symmetry axis. As you know, the sum of the two *focal radii* $PF_1 + PF_2$ is constant for any point P on the ellipse. Explain why this constant equals $2a$. (Hint: Try a special position for P.)



5. (Continuation) Given that $a = 12$ and $c = 9$, find b. (Hint: Try a special position for P.)

6. (Continuation) For any ellipse, what can be said about the quantity $b^2 + c^2$?

Problems for Ellipse Lab – Part III:



7. It is given that P is the point on line AB that makes the sum of distances $CP + PD$ as small as possible. Explain why the angles APC and BPD must be the same size.

8. Two intersecting lines form *four* angles. What can you say about the bisectors of these angles?

9. Suppose that P is a point on an ellipse whose focal points are F_1 and F_2. Draw the intersecting lines PF_1 and PF_2, as well as the bisectors of the four angles they form. This problem is about the bisector that does *not* separate F_1 and F_2. Prove the following: Given any point Q *other than* P on this line, the sum $QF_1 + QF_2$ is greater than the sum $PF_1 + PF_2$. Conclude that the line intersects the ellipse *only* at P.

10. The point P = (6, 5) is on the ellipse $5x^2 + 9y^2 = 405$. Verify this and make a sketch. Then find an equation for the line that intersects the ellipse tangentially at P.

Problems for Hyperbola Lab:

11. Pat and Kim are talking on the telephone during a thunderstorm. After one of the lightning flashes, Pat hears the rumble of thunder *twice* – the first sound coming through the open window, and the second sound coming *through the telephone* ten seconds later. Given that Pat lives two miles east of the center of town, Kim lives two miles west of the center of town, both on the same east-west road, and that sound takes five seconds to travel a mile through air, draw a map that shows some of the places where the lightning could have struck. For example, could the lightning have struck the road on which Pat and Kim live? Assume that light and electricity take no time to reach their destinations.

12. (Continuation) Knowing the additional information that fifteen seconds elapsed between the lightning flash and the first sound of thunder, Pat reasons that there is only *one* place where the lightning could have struck. What is the reasoning? Could Kim have figured all this out?

13. (Continuation) Which equation, $LK – LP = 2$ or $LP – LK = 2$, best fits the situation that you have been investigating?

Problem for Parabola Lab:

14. Let ℓ be the line $y = 1$ and F be the point (–1,2). Verify that the point (2,6) is equidistant from ℓ and F. Sketch the configuration of *all* points P that are equidistant from F and ℓ. This curve is called a *parabola*. The point F is called the *focus* of the parabola, and the line ℓ is called the *directrix*. Find an equation that says that $P = (x,y)$ is on the parabola.